Experimental and Numerical Investigation of Static and Dynamic Analysis of Delaminated Composite Plate

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Experimental and Numerical Investigation of Static and Dynamic Analysis of Delaminated Composite Plate

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This is to certify that the work presented in this dissertation entitled "*Experimental and Numerical Investigation of Static and Dynamic Analysis of Delaminated Composite Plate*" by "*Sushree Sasmita Sahoo*", Roll Number 613ME1013, is a record of original research carried out by her under my supervision and guidance in partial fulfillment of the requirements of the degree of *Master of Technology* in *Mechanical Engineering*. Neither this dissertation nor any part of it has been submitted for any degree or diploma to any institute or university in India or abroad.

Subrata Kumar Panda

Dedicated to……

My Family

Declaration of Originality

I, Sushree Sasmita Sahoo, Roll Number 613ME1013 hereby declare that this dissertation entitled "*Experimental and Numerical Investigation on Static and Dynamic Analysis of Delaminated Composite Plate*" represents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, it contains no material previously published or written by another person, nor any material presented for the award of any other degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the section "Bibliography". I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

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Abstract

Laminated composites are extensively used in various engineering industries like aerospace, civil, marine, automotive and other high performance structures due to their high stiffness to weight and strength to weight ratios, excellent fatigue resistance, long durability and many other superior properties compared to the conventional materials. Delamination is an insidious kind of failure, without being distinct on the surface which causes the layers of a laminated composite plate to detach. It is well known that the propagation of delamination is not only serious but also major concern for the designers in connection to the structural safety. Hence the presence of such defect has to be detected in time to plan the remedial action well in advance. The study aims to analyse mathematically the static, free vibration and transient behaviour of laminated structure with and without delamination by developing numerical models in MATLAB environment based on the higher order shear deformation theory in conjunction with finite element method. Also, a simulation model was developed using finite element software, ANSYS 15.0 and was used for analysis purpose. The static and dynamic analysis of laminated and delaminated plates has also been extended for an experimental validation. Three point bend test via UTM INSTRON 5967 and Modal test via PXIe-1071 was carried out on laminated and delaminated composite specimens and their responses were validated with those of numerical models and simulation model. Based on different numerical illustrations, the effectiveness and the applicability of the presently developed higher order models has been emphasised.

Keywords: *Laminated composite plates; Delamination; Higher-order; Static; FEM; Free Vibration; Transient; MATLAB; ANSYS; Experiment.*

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Nomenclature

(x, y, z)	=	Cartesian coordinate axes
(u, v, w)	=	displacements of any point along the *x, y* and *z*-direction
(u_0, v_0, w_0)	=	displacements of a point on the mid-plane of the panel along *x, y* and *z*-direction
θ_x, θ_y	=	rotations with respect to y and x-direction
$\phi_x, \phi_y, \lambda_x, \lambda_y, \theta_z$	=	higher order terms of Taylor series expansion
U	=	total strain energy
V	=	total kinetic energy
W	=	work done
a, b and h	=	length, breadth and thickness of composite panel
R	=	radius of curvature
E_l, E_t and E_z	=	Young's modulus in the respective material direction
E_{45}	=	Young's modulus in a direction inclined at an angle of 45^0 to the *x* axes
G_{12}, G_{23} and G_{13}	=	Shear modulus in their respective plane (*xy*, *yz* and *xz* plane)
υ_{12}, υ_{23} and υ_{13}	=	Poisson's ratio in their respective direction (*xy*, *yz* and *xz* plane)
ρ	=	Density

Chapter 1

Introduction

1.1 Overview

Since the last three decades, composite materials have been the dominant evolving materials that have penetrated and conquered the vast markets of aerospace, shipbuilding, civil, and mechanical, structural applications relentlessly. Composites are known for their incredibly lightweight and it is equivalent to 25% the weight of steel and 30% lighter than aluminium. The laminated structures are well known for their high strength-to-weight and stiffness-to-weight ratios. They are also resistant to chemicals and will never rust or corrode. They have excellent elastic properties, and some are non-conductive which makes them a good choice for covering electrical industries as well. Innovative designs which were previously impractical can be achieved with the help of composites and no harm in performance or strength. They are capable of deforming due to their flexibility and spring back to their original shape without major damage. It is because of these exceptional features it has led to the steady growth in the volume and the number of applications of the composite materials.

The most common mode of damage in the composite structures is delamination or inter-laminar debonding. The manufacturing process, repeated cyclic stresses, impact during their service life and so on can cause layers of composite laminates to separate which is termed as delamination. The delamination is an insidious kind of failure as it develops inside of the material, without being clear on the surface. It leads to significant loss of mechanical toughness, compressive strength, and cause material unbalance in a symmetric laminate. Due to this, the stiffness of the structure also reduces significantly which later on reduces the natural frequency of the structure. This reduction in the natural frequency may lead to resonance if the resulted value is close to the working frequency value. Such damage becomes a hindrance to the further wide-ranging usage of composite materials. Therefore, the monitoring of internal or hidden damage in the composite material is a critical issue in engineering practice.

While non-destructive testing (NDT) techniques like ultrasonic inspection, mechanical impedance, etc. have been utilized for delamination assessment in composite laminates, they cannot be used for real-time damage detection. Moreover, most of these techniques are mainly labour intensive, time-consuming, and cost ineffective when large structures are measured. This leaves room for static and dynamic assessment as a tool to identify and detect delamination damage and their impact on the structural performance during the service life. Theoretical assessment is quite convenient by employing analytical/numerical methods. Moreover to solve the complex problems, various approximate techniques such as finite difference method, finite element method (FEM), mesh-free method, etc. can be utilised. Out of which, FEM has been dominated the engineering computations since its invention and also expanded to a variety of engineering fields.

1.2 Literature review

Investigation on static, free vibration and transient behaviour of the laminated and the delaminated composite plate have been continuing from last three decades to come up with new and/or modified mathematical models as well the solution steps to overcome the drawbacks of former researches. Numerical and analytical methods have been utilised by the former researchers to bridge the knowledge gap. Laminated composite plates are modelled based on various theories to capture the true structural responses under the influence various types of combined loading. With respect to this, various 2D and 3D models have been proposed for the structural analysis of laminated composite plates. These models are advantageous for the assessment of composite structures, but the computational cost is quite high. This led to the development of an equivalent single-layer (ESL) theory which is derived from the 3D elasticity theory by reducing any 3D problem to a 2D problem. Various ESL theories have been developed in past for the analysis of composite plate are mainly based on the three theories:

- The classical laminate plate theory (CLPT)
- The first-order shear deformation theory (FSDT)
- The higher order shear deformation theory (HSDT)

The classical laminate plate theory and the first-order shear deformation theories have been used by many researchers in past due to their many advantages, and these theories are also encountered with one major lacuna like shear correction factor when analysing for thin laminated structure. Hence, to overcome from the lacuna on the above-discussed theories higher-order theories have been developed subsequently to model the mid-plane kinematic deformations correctly. However, the stress resultants involved in that are difficult to interpret physically and need much more computational effort. In first-order shear deformation theory and higher-order shear deformation theory, the effect of transverse shear deformation may be essential in some cases whereas it is neglected in classical laminate plate theory due to the Kirchhoff hypothesis. The CLPT is based on the Kirchhoff hypothesis that straight lines normal to the undeformed mid-plane remain straight and normal to the deformed midplane and do not undergo stretching in the thickness direction and can be seen in Figure 1.1.

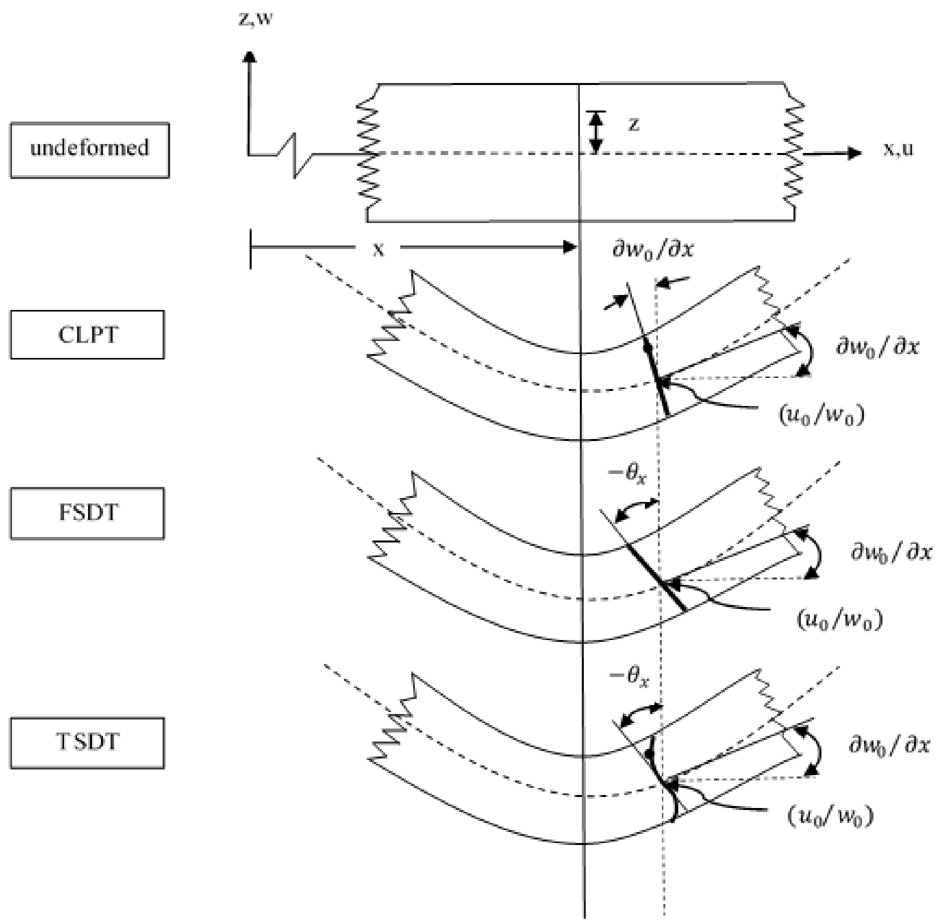


Figure 1.1: Deformation of a transverse normal according to the classical, first-order, and third-order plate theories.

The shell theory used for the present analysis is based on the HSDT mid-plane kinematics model as developed by Reddy in 1984 to analyse composite structures. This theory in integration with FEM is popularly used for laminated structures to obtain global responses like maximum deflection, vibration frequencies, transient response etc. with less computational effort. The higher-order polynomials are used to represent the displacement components through the in-plane and thickness of the plate, and the actual transverse strain/stress through the thickness. It is possible to expand the displacement field in terms of the thickness coordinate up to any desired degree. The reason for expanding the displacements up to the cubic term in the thickness coordinate is to have a quadratic variation of the transverse shear strains and transverse shear stresses through each layer. This avoids the need for shear correction coefficients used in the first-order theory. However, it is important to mention that the complexities in the formulation and large computational effort make it economically unattractive. In addition to that the introduction of the digital computer along with its capability of exponentially increasing computing speed has made the analytically difficult problems amenable to the various numerical methods and thus making the literature rich in this area. In this present study an equivalent single layer model has been considered and it is also assumed that no slippage takes place between the layers. In addition two models has been proposed and developed by taking the transverse displacement is either constant or linear through the thickness. The thickness coordinate of the shell is small compared to the principal radii of curvature.

A detailed evaluation of different issues has been done based on the problems discussed in the above paragraphs. The studies are even enhanced every now and then for more accurate and realistic prediction. Some of the important and selective literature has been discussed here to define the knowledge gap on the available published literature in the subsequent steps. The literature review has been subdivided into two major categories such as static and dynamic analysis of laminated and delaminated composite plate.

1.2.1 Static and Dynamic Analysis of Laminated Composite Plate

In this section, the static, free vibration and transient analysis of laminated structures have been discussed based on the available literature. It is well known that the static and dynamic behaviour of laminated structures is one of the important concerns of many researchers for the prediction and design of structures using available and developed theories. Reddy et al. [1] developed a simulation model of simply supported symmetric laminated composite plate in ANSYS 10.0 to analyze the effect of various parameters on a deflection and stress behavior under uniformly distributed load. Catangiu et al. [2] reported a bending fatigue experimental study for woven glass fiber composites with a predefined defect. Analysis of static, free vibration and buckling behavior of laminated composite and sandwich plates was completed by Cetkovic and Vuksanonic [3] using a generalized layerwise displacement model. Nobakhti and Aghdam [4] reported bending solution of moderately thick rectangular plates resting on two-parameter elastic foundation using generalized differential quadrature method based on the FSDT mid-plane kinematics. Aagaah et al. [5] analysed deformation behaviour of rectangular multi layered laminated composite plate under mechanical loads by deriving the dynamic equations based on a third order shear deformation (TSDT) plate theory in conjunction with von-Karman strains. Kheirikhah et al. [6] presented bending responses of shape memory alloy wire embedded sandwich plate using 3D FEM for flexible core and stiff face sheets. Li et al. [7] reported static and free vibration behavior of several isotropic and laminated composite plates using non-uniform rational B-splines based isogeometric approach based on the TSDT kinematics. Kumar et al. [8] investigated stress and transverse deflections of the laminated composite skew plate with an elliptical hole under pressure loading via FEM using 3D elasticity approach. Thinh and Quoc [9] examined bending failure and free vibration of laminated stiffened glass fiber/polyester composite plates with laminated open section and closed section of stiffeners experimentally and also by FEM using 9-noded isoparametric element with nine degrees of freedom per node. A C^0-type higher-order theory is established by Zhen and Wanji [10] for static analysis of laminated composite and sandwich plates subjected to thermal/mechanical loads. Pandya and Kant [11] reported 3D state of stress, strain and deflection of orthotropic plate based on higher-order displacement model in conjunction with FEM. Viola et al. [12] examined static behavior of doubly-curved laminated composite shells using HSDT kinematics. Mantari et al. [13] presented bending and free vibration responses of sandwich and multilayered composite plates and shells through HSDT kinematic model. Raju and Kumar [14] reported analytical solutions of bending responses of laminated composite plate based on the higher order shear displacement model in conjunction with zig-zag function. Goswami [15] investigated 3D state of stress and strains of laminated thick and thin composite plates using C^0 plate bending element formulation and HSDT theory. Kushwaha and Vimal [16] reported FEM based vibration solutions of laminated composite plate. Kalita and Dutta [17] calculated the natural frequencies in different modes for free vibration of isotropic plates using commercial FE package, ANSYS for different aspect ratios and support conditions. Vibration responses of composite plate are investigated by Patil et al. [18] using FE formulation in ANSYS platform. Free vibration behaviour of moderately thick angle-ply and cross-ply laminated composite plate using FE steps and FSDT kinematic model taking into account the effects of rotary inertia was studied by Sharma and Mittal [19]. Ratnaparkhi and Sarnobat [20] examined free vibration responses of woven fibre Glass/Epoxy composite plate experimentally for free-free support conditions and compared with those FE solutions obtained in ANSYS. Senthamaraikannan and Ramesh [21] determined the effects of various shapes (I, box, and channel) on the mode shapes and modal frequencies of carbon/epoxy composite beams experimentally and compared with simulation results computed in ANSYS. Chakravorty et al. [22] utilised the FSDT kinematics in conjunction with finite element model for the analysis of free vibration responses of doubly curved shell panel with the help of an eight noded curved quadrilateral isoparametric element with five degree of freedom per node. Eruslu and Aydogdu [23] performed vibration analysis of square laminated plates with simply supported edges containing random unidirectionally aligned short fibers based on FSDT. Khdeir and Reddy [24] analysed the free vibration behaviour of cross-ply and angle-ply laminated plates using a generalized Levy type solution in conjunction with state space concept and the model is developed through second order shear deformation kinematic model. Aagaah et al. [25] reported natural frequencies of square angle-ply and cross-ply laminated composite plates for different support conditions by deriving the dynamic equations of motions through TSDT plate kinematics. Tu et al. [26] analyzed bending and free vibration behavior of laminated composite and sandwich plates using C^0 isoparametric FE formulation based on refined HSDT mid-plane kinematics. Tornabene et al. [27] presented free vibration responses of thin and thick doubly-curved laminated composite shell panels using a 2D higher-order general formulation for different curvatures. Cugnoni et al. [28] conducted experimental modal test of glass/polypropylene and calculated the modal responses numerically of thin and thick laminated shells based on the FSDT and HSDT kinematics. Kant and Swaminathan [29] reported analytical solutions of natural frequencies of simply supported composite and sandwich plates based on the HSDT. Reddy et al. [30] developed analytical formulations and corresponding solutions of free vibration responses of functionally graded plates using HSDT mid-plane kinematics. Matsunaga [31] computed natural frequency and buckling load parameter of simply supported cross-ply laminated composite plates using a global higher order plate theory. Desai et al. [32] studied free vibration analysis of multi-layered thick composite plates using an accurate, eighteen node, three-dimensional, higher order, mixed finite element model. Kumar et al. [33] investigated free vibration behaviour of laminated composite anti-symmetric cross-ply and angle-ply plates using zig-zag function based HSDT kinematics. Grover et al. [34] evaluated the free vibration response of laminated composite and sandwich plates by applying recently developed inverse hyperbolic shear deformation. Milan et al. [35] presented FE method solution of the displacement, velocity and acceleration distributions in the unidirectional composite plate consisting of carbon fibers embedded in the epoxy matrix using ANSYS 11.0. Linear static and dynamic analysis of composite plates and moderately thick and thin composite shells were presented by Park et al. [36] considering the quasi-conforming formulation of 4-node stress resultant shell element. Ahmed et al. [37] presented static and dynamic analysis of Graphite/Epoxy composite plates under transverse loading using an eight-noded isoparametric quadratic element based on FSDT with six degrees of freedom at each node. The dynamic behaviour of laminated structural materials with respect to an angular change in fiber layers of the laminated composite was evaluated by Diacenco et al. [38] based on FSDT employing a rectangular element serendipity containing eight nodes. Maleki et al. [39] modelled the transient response of laminated plates with various loading and combinations of clamped, simply supported, and free boundary conditions employing generalized differential quadrature (GDQ) method based on FSDT. The dynamic response of angle-ply laminated composite plates traversed by a moving mass or a moving force was presented by Ghafoori and Asghari [40] using FE method based on FSDT. Marianović and Vuksanović [41] studied the dynamic response under diverse types of dynamic loading of laminated composite plates assuming a layerwise linear variation of displacements components using Reddy's generalized layerwise plate theory (GLPT). Mallikarjuna and Kant [42] examined dynamic behavior multi-layer symmetric composite plate using simple isoparametric finite element formulation based on the HSDT mid-plane kinematics. Makhecha et al. [43] studied the dynamic response of thick multi-layered composite plates using a new HSDT which accounts the realistic variation of in-plane and transverse displacements through the thickness. Mantari et al. [44] studied the static and dynamic response of spherical, cylindrical shells and sandwich plates with simply supported boundary conditions subjected to bi-sinusoidal, distributed and point loads using HSDT. Raju and Kumar [45] presented transient analysis for anti-symmetric cross-ply and angle-ply laminated composite plates subjected to mechanical loading based on higher order shear displacement model with zig-zag function.

1.2.2 Static and Dynamic Analysis of Delaminated Composite Plate

As mentioned earlier, the presence of delamination in laminated composite plates influences the static and dynamic behaviour drastically. From a design point of view, the prediction of delamination is of paramount importance for the researchers. The static, free vibration and transient analysis of delaminated composite structures based on the available literature have been discussed in this section. Nanda [46] calculated the flexural deformations of delaminated composite shell panels using a layerwise theory and associated FE model based on the assumption of FSDT. Sekine et al. [47] analysed the buckling characteristics of elliptically delaminated composite laminates by considering partial closure of the delamination. Ovesy et al. [48] examined the effects of delamination on the dynamic buckling behaviour of a composite plate with delamination by implementing semi analytical finite strip method. The effect of multiple impact induced delaminations upon the buckling response of laminated structures with multiple circular and/or elliptic delaminations was investigated by Obdrzalek and Vrbka [49] by the means of the finite element analysis. Parhi et al. [50] evaluated the first ply failure of laminated composite plates with randomly located multiple delaminations subjected to transverse static and impact by applying finite element analysis procedure using the FSDT via eight-node isoparametric quadrilateral elements. Mohanty et al. [51] studied the parametric instability characteristics of delaminated composite plates subjected to periodic in-plane load by developing a FE method based on the assumptions of FSDT. Szekrenyes [52] applied the TSDT to calculate the stresses and energy release rates in delaminated orthotropic composite plates with straight crack front. By means of FE approach, the effects of localized interface progressive delamination on the behavior of two-layer laminated composite plates was studied by Sabah and Kueh [3] when subjected to low velocity impact loading for various fibre orientations. For the analysis of coupled electromechanical response of composite beams with delamination with embedded piezo actuators and sensors, Chrysochoidis and Saravanos [54] formulated a coupled linear layerwise laminate theory. The natural frequencies of GFRP delaminated circular plates were investigated by Hou and Jeronimidis [55] numerically through the commercial FE package (ALGOR) and experiment. The concept of continuous analysis was adopted by Chang et al. [56] to model a delaminated composite plate and analysed the free vibrations of the plate when it is subjected to an axial load. Sultan et al. [57] investigated the natural frequencies of the simply supported composite laminate plates with different areas of mid plane delamination experimentally and numerically by the application of FE analysis software ANSYS 12.1. Similarly, Al-Waily [58] fabricated woven composite plate and calculated its natural frequency by using experimental work for various aspect ratio and boundary conditions of plate and compared them with numerical study. Kumar et al. [59] analysed the free mode vibration of composite laminates with delamination by employing a nine noded quadrilateral MITC9 element and compared the results obtained with the available experimental and 3D FE simulations. Alnefaie [60] studied the natural frequencies and modal displacements are calculated for various delaminated fibre-reinforced composite plates with different dimensions and delamination characteristics by employing a 3D FE model. The impact of delamination on the natural frequencies of composite plates, as well as delamination dynamics over a broad frequency range was studied by Tenek et al. [61] using the FEM based on the three-dimensional theory of linear elasticity. Ju et al. [62] computed the natural frequencies and mode shapes of composite laminated plates with multiple delaminations based on Mindlin’s plate theory. Based on the same theory, Day and Karmakar [63] employed FE method to inspect the effects of delamination on free vibration characteristics of Graphite-Epoxy pretwisted shallow angle-ply composite conical shells. Natural frequency, modal displacement and modal strain are analysed for multi layered composite plates with different dimensions of delamination by Yam et al. [64] experimentally as well as numerically by employing a 3D FE model. Nanda and Sahu [65] applied a FE formulation based on FSDT to investigate the natural frequencies and mode shapes of composite shells with multiple delaminations. Based on FSDT again, Panda et al. [66] investigated the free vibration behaviour of woven fibre Glass/Epoxy delaminated composite plates subjected to hygrothermal environment numerically and experimentally. Then again, for elevated temperature and moisture content and FSDT assumptions, a quadratic isoparametric FE formulation was presented by Parhi et al. [67] for the free vibration and transient response analysis of multiple delaminated doubly curved composite shells. The effect of lamination angle, location, size and the number of delamination on free vibration frequencies of a delaminated composite beam using one-dimensional layerwise FE model were studied by Lee [68]. HSDT mid-plane kinematics was used to compute natural frequency, critical buckling load and instability regions of a Graphite/Epoxy composite plate with and without delamination by Chattopadhyay et al. [69]. The free vibration responses of a sandwich plate with anisotropic composite laminated faces were found out by Hu and Jane [70] when applied to an axial load. Della and Shu [71] reviewed numerous analytical models and numerical analyses for the free vibration of delaminated composite laminates classified according to the fundamental theory and also presented a comparison of some of the results of these models. Meimaris [72] proposed a twenty noded, isoparametric, parabolic, solid element for modelling laminated anisotropic plates for its free vibration and transient analysis. The transient response of a composite plate with a near-surface delamination was studied by Zhu et al. [73]. Marjanovic [74] proposed a computational model based on the Generalized Laminated Plate Theory for the transient response of laminated composite and sandwich plates having zones of partial delamination when subjected to dynamic pulse loading. Parhi et al. [75] studied the dynamic behaviour of multi layered laminated composite plates having arbitrarily located multiple delamination by deriving equations based on eight noded isoparametric quadratic element in the framework of FSDT. Chandrashekhar and Ganguly [76] applied Reddy's third order theory with C^0 assumed strain interpolation to analyse large deformation dynamic response of delaminated composite plates. Chattopadhyay et al. [77] employed a FE model based on HSDT to investigate the dynamic instability associated with composite plates with delamination when subjected to dynamic loading. Huang et al. [78] studied the hygrothermal effects on the nonlinear vibration and dynamic response of shear deformable laminated plates with the assumptions HSDT mid-plane kinematics. The dynamic instability associated with composite delaminated plates subjected to dynamic compressive loads was investigated by Radu and Chattopadhyay [79] by applying a refined HSDT. A higher-order cubic zigzag theory was proposed by Oh et al. [80] to analyse the dynamic behaviour of a laminated composite plate with multiple delaminations.

1.3 Introduction of Finite Element Method and ANSYS

Based on the above-discussed literature, it is realized that there is a strong need of a generalized mathematical model which can predict the structural responses, i.e.- static, free vibration and transient responses of the laminated composite plate with and without delamination, precisely under the combined thermo-mechanical loading. To address the same, majority of research of laminated and delaminated plates have been carried out based on the theoretical analysis by involving analytical and/or numerical methods. It is true that the closed-form solution provides the exact responses of the structure, but these are limited to the simple problems only. Therefore, to solve the complex problems, various approximate techniques such as finite difference method, finite element method (FEM), mesh-free method, etc. have been utilised in past to evaluate the desired responses by incorporating the real-life situations. Out of all approximated analysis, the FEM has been dominated the engineering computations since its invention and also expanded to a variety of engineering fields. FEM is widely adopted and being used as the most trustworthy tool for designing of any structure because of the higher accuracy of this method compare to other analytical or numerical methods. It plays an important role in predicting the responses of various products, parts, assemblies and subassemblies. Use of FEM in all advanced industries saves huge time of prototyping with reducing the cost due to physical test and increases the innovation at a faster and more accurate way. In this regard, two generalized mathematical models are proposed to be developed based on the HSDT mid-plane kinematics employing a nine noded isoparametric element with nine and ten degrees of freedom for the static and dynamic analysis of laminated and delaminated composite plates.

There are many optimized finite element analysis (FEA) tools are available in the market and ANSYS is one of them which is accepted by many industries and analysts. ANSYS is being used in a different engineering fields such as power generation, electronic devices, transportation, and household appliances as well as to analyse the vehicle simulation and in aerospace industries. ANSYS gradually entered into a number of fields making it convenient for fatigue analysis, nuclear power plant and medical applications. In the present work, the static and dynamic analysis of composite laminated and delaminated plates is done by taking shell element SHELL281 from the ANSYS library. It is an eight noded linear shell element with six degrees of freedom at each node which includes translation motion in x, y, z direction and rotation motion about x, y, z axis. It is well-suited for thin to moderately thick shell structures and linear, large rotation, and/or large strain nonlinear applications.

1.4 Motivation of the Present Work

The laminated composite plates are of great attention to the designers because of efficient lightweight structures, due to their numerous advantageous properties as mentioned earlier in Section 1.1. The increased intricate analysis of the composite structures is especially due to its greater use in the field of aeronautical/aerospace engineering. These structural components are subjected to various types of combined loading in their service life which leads to the separation of the layers of laminated plates which is characterized as delamination. Presence of delamination in the laminated plate disturbs the static, free vibration and transient responses significantly. Therefore, a thorough behavioural study of the delaminated composite structure has to be examined.

As discussed earlier, non-destructive testing (NDT) techniques like ultrasonic inspection, mechanical impedance etc. have been applied to accurately assess the effect of delamination composite laminated plate. But these methods cannot be used for real time damage detection. Moreover, most of these techniques are mainly labour intensive, time consuming, and cost ineffective when large structures are measured. Hence, to investigate the structural responses, the numerical approaches can be implemented especially when the material, the geometry and the loading types are complex in nature. Therefore, a general mathematical model has to be developed which can compute the true static, free vibration and transient responses of the laminated composite plates with delamination.

It is evident from the exhaustive literature review that most of the previous studies are focused to address on the issues related to the static, free vibration and transient responses of the laminated and/or the delaminated composite structures using the FSDT kinematics in conjunction with FEM and very few utilised the HSDT kinematics. It is also noted that no study has been reported yet on the static, free vibration and the transient responses of the laminated and delaminated composite using two higher-order kinematics with subsequent validation with simulation and experimental results.

1.5 Objectives and Scope of the Present Thesis

The main purpose of the work is to develop general mathematical model for the laminated composite plate with seeded delamination to investigate the static, free vibration and transient analysis. In this regard, two higher order models, Model-1 and Model-2, are proposed and developed based on the assumptions of the HSDT mid-plane kinematics employing a nine noded isoparametric Lagrangian element having nine and ten degrees of freedom per node, respectively. In addition to that, a simulation model namely, Model-3 is also developed in commercial FE package (ANSYS 15.0) based on ANSYS parametric design language (APDL) code to compute the desired responses. The study is further extended to analyse the structural responses of laminated plates having delamination employing the higher order models. Experimental studies were also carried out to analyse the static and free vibration responses of laminated and delaminated composite plates and their responses were validated with the developed models. The study also aims to obtain the effect of different geometrical parameters (aspect ratios, thickness ratios and modular ratios) and support conditions on the static, free vibration and transient responses of both laminated and delaminated composite plates. The point-wise description of the scope of the present study is discussed below:

- As a first step, the static responses of laminated composite plate considering various geometrical and material properties has been studied using the proposed numerical and simulation models with the help of computer code developed in MATLAB and ANSYS 15.0 environment, respectively.
- The models are extended to study the dynamic responses, i.e., the free vibration and transient responses of laminated composite plates through the developed numerical and simulation models.
- Further, the higher-order models are extended to analyse the static responses of delaminated composite plates.
- The dynamic responses of the delaminated composite plates are analysed using the same steps by utilizing the proposed higher-order models.
- The static and the dynamic responses of laminated and delaminated plate have also been extended for the experimental validation. Three-point bend test and modal test are carried out on laminated and delaminated composite specimens, and their responses are compared with those of developed models.
- Finally, the parametric study of laminated and delaminated composite plates has been carried out using the developed higher order models.

Few numerical examples of laminated composite plates with and without delamination are solved using the proposed models. In order to check the efficacy of the present developed mathematical models and simulation models as well, the convergence behaviour with mesh refinement has been computed for the composite plates. In addition to that, the present static and dynamic responses of laminated and delaminated composite plates are also compared with those available published literature and experimental results to show the accuracy of the presently developed numerical models. Finally, a good volume of new results are computed for the future references in this field of study.

1.6 Organisation of the Thesis

The overview and motivation of the present work followed by the objectives and scope of the present thesis are discussed in this chapter. The background and state of the art of the present problem by various investigators related to the scope of the present area of interest are also addressed in this chapter. This chapter divided into five different sections, the first section, a basic introduction to problem and theories used in past. In section two, as there is need to know the state of the art of the problem, earlier work done in the same field has been reviewed exhaustively. Hence, some important contributions to the static and the dynamic behaviour of the laminated and delaminated composite plate are highlighted in the second section. Subsequently, a brief introduction of finite element method and finite element analysis software, ANSYS is put forward in the third section. The motivation and the main objective and scope of present work are discussed in the fourth and fifth section, respectively. Some of the critical observations are discussed in the final section. The remaining part of the thesis are organised in the following fashion.

In Chapter 2, the general higher-order mathematical model development for the static, free vibration and transient analysis of the laminated and the delaminated composite structure are discussed. In continuation to that, the simulation model development using commercial simulation software of the said problem is also discussed. In order to achieve the structural responses, the kinematic field, the constitutive relations, the finite element formulations, and the solution techniques that are required for the analysis purpose are detailed in this chapter. Subsequently, the support condition and computational investigation are discussed. In addition, the detailed regarding the computational implementation of the presently developed mathematical model is also discussed in details.

Chapter 3 illustrates the static, free vibration and dynamic responses of laminated composite plates. Detailed parametric studies of material and geometrical parameters are also discussed. Subsequently, the outcomes are summarised in the conclusion section.

The static, free vibration and dynamic responses of delaminated composite plates are highlighted in Chapter 4. Also, the thorough parametric studies of material and geometrical parameters and effect of size and location of delamination are discussed. Based on the results, the concluding remarks are also outlined.

Chapter 5 summarizes the whole work, and it contains the concluding remarks drawn from the present study and the future scope of the work of the present study.

Chapter 2

General Mathematical Formulation

2.1 Introduction

In today's modern world, many weight sensitive industries are using high performance laminated composite structures and their components. These structural components very often tend to deflect due to excessive loading conditions during their service life. Hence, these components must be reliable enough and should have good load bearing capacity. Also, the manufacturing process, repeated cyclic stresses, impact, and unlike environment condition may cause layer separation of laminated composites which is known as delamination. Delamination is an insidious kind of failure as it develops inside of the material, without being clear on the surface. Delamination leads to significant loss of mechanical toughness, compressive strength, and cause material unbalance in a symmetric laminate which becomes a hindrance to the further wide-ranging usage of composite materials. Therefore, the monitoring of internal or hidden damage in composite material is critical issue in engineering practice. While non-destructive testing (NDT) techniques like ultrasonic inspection, mechanical impedance etc. have been utilized for delamination assessment in composite laminates, they cannot be used for real time damage detection. This leaves room for static and dynamic assessment as a tool to identify and detect delamination damage and their impact on the structural performance during service life. Also, the laminated composite structural components are extremely flexible as compared to other metallic components so the large deformation terms and the higher order shear deformation terms arises during the mathematical modelling are more important for an accurate prediction of the central deflection and natural frequency.

A thorough literature review in the previous chapter clearly shows that various studies have been done previously on the layered structures for different types of analyses. But very few studies have been reported on the static and dynamic behaviour of laminated composite plate by taking two models in the framework of the HSDT kinematics and subsequent validation with those available published literature, experiments and simulation model developed in commercial FE package (ANSYS) using ANSYS parametric design language (APDL) code.

In this chapter a general mathematical formulation of laminated composite plate is developed on the basis of basic assumptions. The system of governing equations for the static free vibration and transient response of laminated composite plates are derived using the variational approach, Hamilton's principle and Newmark's integration scheme respectively. Three finite element models based on existing HSDT kinematics in conjunction with isoparametric FEM steps are employed to discretise the present model and to compute the desired responses numerically.

2.2 Assumptions

The mathematical formulation is based on the following assumptions:

(i) The fibres and matrices of the laminated composite plate are assumed to be in perfect bond and no slippage occurs at the interface.
(ii) All the layers of the laminated plate are assumed to be bonded together where each layer is treated as homogeneous and orthotropic.
(iii) The reference plane is assumed to be at the mid-plane of the plate.
(iv) A two dimensional approach has been implemented to model a three dimensional response of composite plate.
(v) For the present analysis the material properties are considered in ambient conditions.
(vi) The composite properties are assumed to be independent of temperature and moisture.

2.3 Mathematical model for plate without delamination

In this analysis, a laminated composite plate composed of the N number of orthotropic layers of uniform thickness h, length a, and width b is considered as shown in Figure 2.1. The displacement field kinematics within the laminate is assumed to be based on the HSDT/FSDT, where the inplane displacements are expanded as cubic/linear functions of the thickness coordinate while the transverse displacement varies either linearly and/or constant through the plate thickness. In this study, three different kinematic models have been utilized for the analysis purpose and discussed in the following lines:

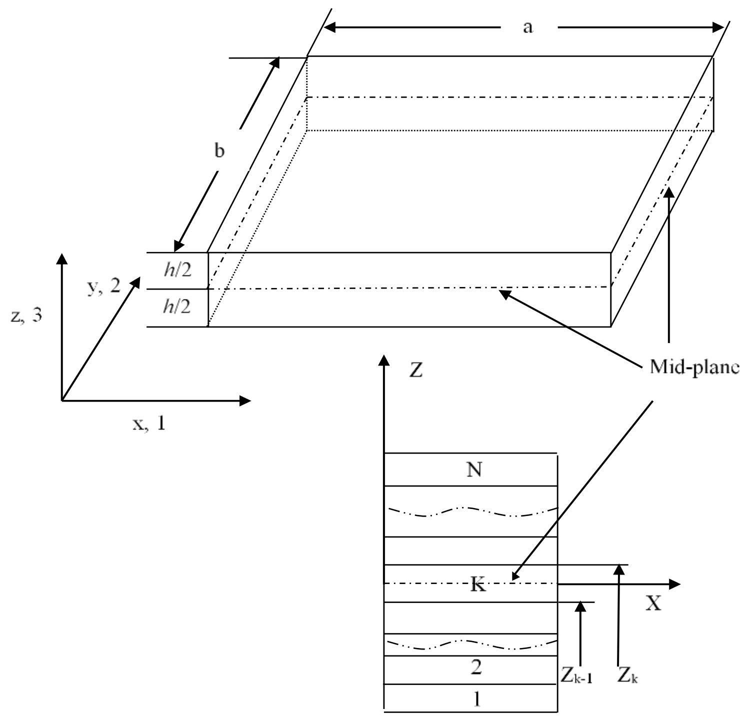


Figure 2.1: Laminated composite plate geometry with stacking sequence

2.3.1 Model-1

The mathematical model of the laminated composite plate is developed based on the HSDT kinematics as in Reddy [81] considering the transverse displacement is constant through the thickness and it results in zero transverse normal strain. The displacement filed is conceded as:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y) + z\theta_x(x,y) + z^2\phi_x(x,y) + z^3\lambda_x(x,y) \\ v(x,y,z,t) &= v_0(x,y) + z\theta_y(x,y) + z^2\phi_y(x,y) + z^3\lambda_y(x,y) \\ w(x,y,z,t) &= w_0(x,y) \end{aligned} \tag{2.1.a}$$

2.3.2 Model-2

Similarly, the second model is also developed using another HSDT kinematic model considering the linear variation of the displacement along the thickness direction as in Singh and Panda [82]:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y) + z\theta_x(x,y) + z^2\phi_x(x,y) + z^3\lambda_x(x,y) \\ v(x,y,z,t) &= v_0(x,y) + z\theta_y(x,y) + z^2\phi_y(x,y) + z^3\lambda_y(x,y) \\ w(x,y,z,t) &= w_0(x,y) + z\theta_z(x,y) \end{aligned} \tag{2.1.b}$$

2.3.3 Model-3

As discussed earlier, a simulation model has been developed in ANSYS using APDL code. For the discretization purpose, Shell 281 element has been chosen from ANSYS element library. The Shell281 is a serendipity eight noded element with six degrees freedom per nodes and capable to solve thin to moderately thick shell structure. The mid-plane kinematics of the simulation model is same as the FSDT variation as in Reddy [81] and conceded as:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z,t) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z,t) &= w_0(x,y) + z\theta_z(x,y) \end{aligned} \tag{2.1.c}$$

where, t is the time and u, v and w are the displacements of any point along the x, y and z coordinate axes respectively. u_0, v_0 and w_0 are corresponding displacements of a point on the mid plane and θ_x and θ_y are the rotations of normal to the mid-surface, i.e., z=0 about the y and x-axes, respectively. The functions $\phi_x, \phi_y, \lambda_x, \lambda_y$ and θ_z are the higher order terms in the Taylor series expansion.

2.4 Strain-Displacement Relation

The stress-strain relations for any k^{th} lamina oriented at an arbitrary angle ϕ about any arbitrary axes are given by:

$$\{\sigma\} = \left[\overline{Q}_{ij}\right]\{\varepsilon\} \tag{2.2}$$

where, $\{\sigma\}$, $\left[\overline{Q}_{ij}\right]$ and $\{\varepsilon\}$ are the stress tensor, reduced stiffness matrix, and strain tensor, respectively. The strain matrix for any laminated composite plate can be further written as:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial u}{\partial x}\right) \\ \left(\dfrac{\partial v}{\partial y}\right) \\ \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) \\ \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \end{Bmatrix}$$

(2.3)

The stress tensor further modified in force form as:

$$\{F\} = [D]\ \{\varepsilon\} \tag{2.4}$$

where, [D] is the property matrix and can be obtained using the following steps:

$$[D] = \left[A_{ij}, B_{ij}, D_{ij}\right] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k \left(1, z, z^2\right) dz \ \ (i, j = 1, 2, 6) \tag{2.5}$$

where, $[A_{ij}]$ is the in-plane stress matrix, $[B_{ij}]$ is the coupling stiffness matrix and $[D_{ij}]$ is the bending stiffness matrix.

2.5 Finite Element Formulation

In the current analysis, a nine noded isoparametric quadrilateral Lagrangian element is employed. The details of the element can be seen in Cook et al. [83]. The displacement fields for different assumed kinematic models are expressed in terms of desired field variables and the models are discretised using suitable FEM steps.

The displacement vector '*d*' at any point in the mid-plane for any model is given by

$$d = \sum_{i=1}^{n} N_i\left(x, y\right) d_i \tag{2.6}$$

where, $\{d_i\} = \left\{u_{0_i}\ \ v_{0_i}\ \ w_{0_i}\ \theta_{x_i}\ \ \theta_{y_i}\ \ \phi_{xi}\ \phi_{yi}\ \ \lambda_{xi}\ \ \lambda_{xi}\right\}^T$, $\{d_i\} = \left\{u_{0_i}\ \ v_{0_i}\ \ w_{0_i}\ \theta_{x_i}\ \ \theta_{y_i}\ \theta_{zi}\ \phi_{xi}\ \phi_{yi}\ \lambda_{xi}\ \ \lambda_{yi}\right\}^T$ and $\{d_i\} = \left\{u_{0_i}\ \ v_{0_i}\ \ w_{0_i}\ \theta_{x_i}\ \ \theta_{y_i}\ \theta_{zi}\right\}^T$ are the nodal displacement vectors for Model-1, Model-2, and Model-3, respectively and N_i is the corresponding interpolating functions, associated with the 'i^{th}' node and the details can be seen in Appendix (B.1).

The strain vector can be written in the matrix form and conceded as:

$$\{\varepsilon\} = [T]\{\bar{\varepsilon}\} \tag{2.7}$$

where, [T] is the thickness coordinate matrix whose details are furnished in equation B.2 of appendix and $\{\bar{\varepsilon}\}$ is the mid-plane strain vector which can be further reduced as:

$$\{\bar{\varepsilon}\} = [B_L]\{d\} \tag{2.8}$$

where, $[B_L]$ is the product form of differential operators and shape functions.

2.6 Energy Calculation

As a first step, the global displacement vector can be expressed in matrix form

$$\{\bar{\delta}\} = \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} = [f]\{\delta\} \tag{2.9}$$

where, $[f]$ is the functions of thickness coordinate.

The total strain energy, U of a laminated composite plate can be expressed as:

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T \{\sigma\} dV = \frac{1}{2}\iint \left[\int_{-h/2}^{+h/2} \{\varepsilon\}^T \{\sigma\}\, dz \right] dxdy \tag{2.10}$$

Again, the above equation can be rewritten by substituting equation (2.6) in equation (2.10) as:

$$U = \frac{1}{2}\iint \left(\{\bar{\varepsilon}\}^T [D]\{\bar{\varepsilon}\} \right) dxdy = \frac{1}{2}\iint \left(\{\delta\}_i^T [B_L]_i^T [D][B_L]_i \{\delta\}_i \right) dxdy \tag{2.11}$$

where, $[D] = \int_{-h/2}^{+h/2} [T]^T [Q_{ij}][T]\, dz$

The total kinetic energy, T of a laminated composite plate can be expressed as:

$$T = \frac{1}{2}\int_V \rho \{\dot{\delta}\}^T \{\dot{\delta}\} dV \tag{2.12}$$

where, ρ, and $\{\dot{\delta}\}$ are the mass density and the first order differential of the displacement vector with respect to time, respectively.

Using equation (2.9) and (2.12) for '*N*' number of orthotropic layered composite plate, the kinetic energy equation can be written as:

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{\dot{\delta}\right\}^T \left[f\right]^T \rho^k \left[f\right]\left\{\dot{\delta}\right\} dz \right) dA = \frac{1}{2}\int_A \left\{\dot{\delta}\right\}^T \left[m\right]\left\{\dot{\delta}\right\} dA \tag{2.13}$$

where, $\left[m\right] = \sum_{k=1}^{N} \int_{z_{k=1}}^{z_k} \left(\left[f\right]^T \rho^k \left[f\right]\right) dz$ is the inertia matrix.

The final form for T by substituting Eq. (2.6) in Eq. (2.13) can be written as:

$$T = \int_A \left(\left[N_i\right]^T \left[m\right]\left[N_i\right] dA\right)\left\{\ddot{\delta}\right\} \tag{2.14}$$

The total work done by an externally applied load, F is given by:

$$W = \int_A \left\{\delta\right\}^T \left\{F\right\} dA \tag{2.15}$$

The stiffness matrix ([K]) and the mass matrix ([M]) of the composite plate can be further expressed as in the following line:

$$\begin{aligned} [K] &= \int_A \left(\sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [B_L]^T [D][B_L] dz\right) dA \\ [M] &= \int_A \left(\sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [N]^T [N] \rho dz\right) dA \end{aligned} \tag{2.16}$$

2.7 Mathematical model for the delaminated plate

Now, the earlier developed higher order models are further extended for the plate with delaminations. For this, a typical laminate with *'p'* number of delamination is considered as shown in Figure 2.2. For the delaminated element, the local coordinate system is considered to be $0'$, x', y' and z' which is analogous to the coordinate system used for the segment without delamination. The displacement field of the delaminated element is assumed to be of the following form relative to its own local coordinate system.

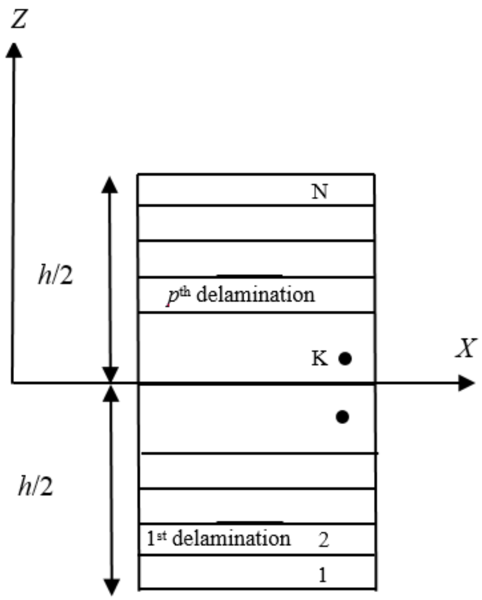


Figure 2.2: Laminated plate with delamination

2.7.1 Model-1

The first model for the delaminated segment of the plate is developed based on the HSDT kinematics similar to the laminated segment considering the transverse displacement is constant through the thickness and it results in zero transverse normal strain. The displacement filed is conceded as:

$$\begin{aligned} u'\left(x',y',z',t\right) &= u_0'\left(x',y'\right)+z'\theta_x'\left(x',y'\right)+z'^2\phi_x'\left(x',y'\right)+z'^3\lambda_x'\left(x',y'\right) \\ v'\left(x',y',z',t\right) &= v_0'\left(x',y'\right)+z'\theta_y'\left(x',y'\right)+z'^2\phi_y'\left(x',y'\right)+z'^3\lambda_y'\left(x',y'\right) \\ w'\left(x',y',z',t\right) &= w_0'\left(x',y'\right) \end{aligned} \quad (2.17.a)$$

2.7.2 Model-2

Also, the second model is developed in a similar fashion considering the linear variation of the displacement along the thickness direction as:

$$\begin{aligned} u'\left(x',y',z',t\right) &= u_0'\left(x',y'\right)+z'\theta_x'\left(x',y'\right)+z'^2\phi_x'\left(x',y'\right)+z'^3\lambda_x'\left(x',y'\right) \\ v'\left(x',y',z',t\right) &= v_0'\left(x',y'\right)+z'\theta_y'\left(x',y'\right)+z'^2\phi_y'\left(x',y'\right)+z'^3\lambda_y'\left(x',y'\right) \\ w'\left(x',y',z',t\right) &= w_0'\left(x',y'\right)+z'\theta_z'\left(x',y'\right) \end{aligned} \quad (2.17.b)$$

where, t is the time and $u^{'}$, $v^{'}$ and $w^{'}$ are the displacements of any point along the $x^{'}$, $y^{'}$ and $z^{'}$ coordinate axes, respectively. $u_0^{'}$, $v_0^{'}$ and $w_0^{'}$ are corresponding displacements of a point on the mid plane and $\theta_x^{'}$ and $\theta_y^{'}$ are the rotations of normal to the mid-surface, i.e., $z^{'}$=0 about the $x^{'}$ and $y^{'}$ axes, respectively. The functions $\phi_x^{'}, \phi_y^{'}, \lambda_x^{'}, \lambda_y^{'}$ and $\theta_z^{'}$ are the higher order terms in the Taylor series expansion.

2.8 Displacement continuity conditions

In the earlier sections, the elements in the segment without delamination and delaminated segment are derived separately, where all the variables in the displacement field are independent of each other. But, at the boundary that connects the segment without delamination with the delaminated segment, the continuity conditions for the displacement field ought to be fulfilled.

Figure 2.3 illustrates the elements at the boundary connecting the undelaminated segment and the delaminated segment. Let element *m*, be the delaminated element in the delaminated segment, whereas element 1 is an element in the undelaminated segment which is the immediate left of the element *m*. The common nodes 1, 4 and 8 are at the boundary that connects both the segments. Let [*k*], [*m*] and $\{d\}$ and [*k'*], [*m'*] and $\{d'\}$ be the elemental stiffness matrix, elemental mass matrix and elemental nodal displacement vector for element 1 and element *m*, respectively.

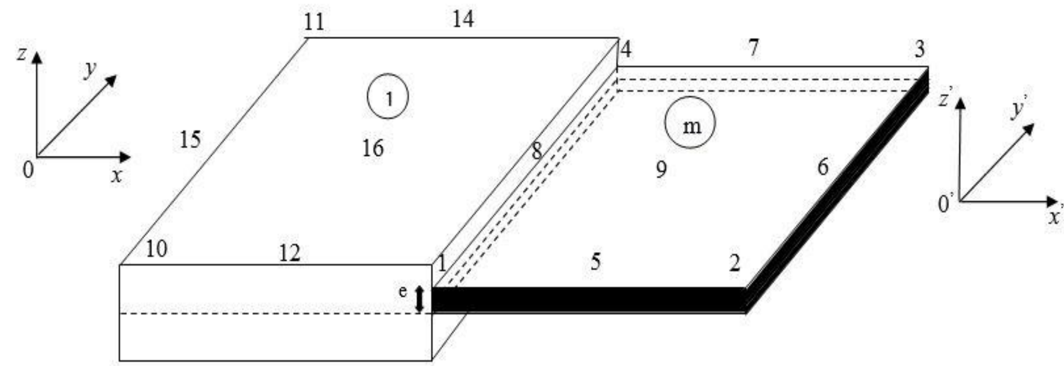


Figure 2.3: Elements at connecting boundary

For the common nodes 1, 4 and 8, let

$$\{d_i\} = \{u_{0_i}\ v_{0_i}\ w_{0_i}\ \theta_{x_i}\ \theta_{y_i}\ \theta_{zi}\ \phi_{xi}\ \phi_{yi}\ \lambda_{xi}\ \lambda_{yi}\}^T \; i\text{=1, 4 and 8 for element 1} \tag{2.19.a}$$

$$\{d_i^{'}\} = \{u_{0i}^{'}\ v_{0i}^{'}\ w_{0i}^{'}\ \theta_{xi}^{'}\ \theta_{yi}^{'}\ \theta_{zi}^{'}\ \phi_{xi}^{'}\ \phi_{yi}^{'}\ \lambda_{xi}^{'}\ \lambda_{yi}^{'}\}^T \quad i\text{=1, 4 and 8 for element } m \tag{2.19.b}$$

where, *e* is the distance between the mid-plane of element 1 and the mid-plane of element *m*, and *e* is always considered to be positive in the sense indicated in Figure 2.3. The present model i.e., Eq. (2.20) is suitable for the analysis of the global response of the laminated structures. However, the local effects (such as the perturbation of the stress field around the delamination front) are not taken into consideration for the present study. The system of exact kinematic conditions (SEKC) makes it possible to consider the local effects for the similar type of analysis.

Therefore,

$$\{d_i\} = [\lambda]\{d_i^{'}\} \qquad \text{for } i\text{=1, 4 and 8} \tag{2.21}$$

Now, using equation (2.19), the following transformation of element *m* can be written as follows:

$$\{d^{'}\} = [T]\{\bar{d}^{'}\} \tag{2.22.a}$$

where,

where, the matrix [*I*] is a 9 × 9 unit matrix for the higher order.

The transformations for the elemental stiffness matrix and elemental mass matrix of the element *m* can be expressed as:

$$\begin{aligned} [\bar{k}'] &= [T]^T [k'][T] \\ [\bar{m}'] &= [T]^T [m'][T] \end{aligned} \tag{2.23}$$

The displacement continuity conditions have been incorporated into $[\bar{k}']$ and $[\bar{m}']$, and the nodal displacement $\{d_i'\}$ is replaced by $\{d_i\}$ for i = 1, 4, 8. The matrices $[\bar{k}']$ and $[\bar{m}']$, are subsequently used to assemble the global stiffness matrix and global mass matrix, respectively. In addition, a similar treatment can be carried out for the cases where the connecting boundaries of the two segments are at the other edges.

2.9 Governing Equations

2.9.1 Static analysis

The final form of governing equation for bending analysis of laminated plate is obtained using variational principle:

$$\delta \prod = \delta U - \delta W = 0 \tag{2.24}$$

where, δ is the variational symbol and $\prod$ is the total potential energy.

The equilibrium equation for the bending analysis is obtained by substituting equations (2.6), (2.11) and (2.15) into equations (2.24) and expressed as following:

$$[K]\{d\} = \{F\} \tag{2.25}$$

2.9.2 Free vibration analysis

Now, the final form of governing equation for free vibrated laminated plate is obtained by using Hamilton's principle and expressed as:

$$\delta \int_{t_1}^{t_2} (T - U)dt = 0 \tag{2.26}$$

Substituting equations (2.6), (2.14) and (2.15) into equation (2.26), the final form of the equation will be conceded as:

$$[M]\{\ddot{d}_i\}+[K]\{d_i\}=0 \tag{2.27}$$

where, $\ddot{d}_i$ is the acceleration and d_i is the displacement vector, respectively.

Further, the eigenvalue form of the governing equation is obtained by dropping the appropriate term to compute the natural frequency and the system equation is conceded as:

$$\left([K]-\omega^2[M]\right)\Delta=0 \tag{2.28}$$

where, ω and Δ are the natural frequency and the corresponding eigenvector, respectively.

2.9.3 Transient analysis

In transient analysis, at a particular time *t*, the static equilibrium along with the effects of acceleration-dependent inertia forces and velocity-dependent damping forces are considered. The equation of equilibrium governing the linear transient response of a system is expressed as follows:

$$[M]\ddot{d}+[C]\dot{d}+[K]d=[F] \tag{2.29}$$

where, $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $[F]$ is the external force vector and $\ddot{d}$, $\dot{d}$ and d are the acceleration, velocity and displacement vectors, respectively. Now, the transient equation is solved for few time step Δt within the whole time period of *T*, using the constant average acceleration steps of Newmark's integration scheme. The desired transient equation is obtained using the Newmark's integration parameters such as α, δ and a_0 to a_7 and the details are provided in appendix (B.3) as same as Bathe [84]. For a particular time step *t*, the effective stiffness matrix is expressed as follows:

$$[\hat{K}]=[K]+a_0[M] \tag{2.30}$$

Similarly, the effective load matrix for the successive time step $t+\Delta t$ is taken to be:

$${}^{t+\Delta t}[\hat{F}]={}^{t+\Delta t}[F]+[M](a_0\,{}^{t}d+a_2\,{}^{t}\dot{d}+a_3\,{}^{t}\ddot{d})+[C](a_1\,{}^{t}d-a_4\,{}^{t}\dot{d}+a_5\,{}^{t}\ddot{d}) \tag{2.31}$$

Now the desired displacement, acceleration and velocity from the transient analysis can be computed using the following equations:

$$\left.\begin{aligned} &[\hat{K}]^{t+\Delta t}d = {}^{t+\Delta t}[\hat{F}] \\ &{}^{t+\Delta t}\ddot{d} = a_0({}^{t+\Delta t}d - {}^{t}d) - a_2\,{}^{t}\dot{d} - a_3\,{}^{t}\ddot{d} \\ &{}^{t+\Delta t}\dot{d} = {}^{t}\dot{d} + a_6\,{}^{t}\ddot{d} + a_7\,{}^{t+\Delta t}\ddot{d} \end{aligned}\right\} \tag{2.32}$$

2.10 Boundary Conditions

In order to avoid the rigid body motion as well as to reduce the number of unknowns of a system, the system composite plate is applied to different boundary condition. Boundary condition further eases the calculation and also avoids the singularity in the matrix equation. In this work, only the displacement variables are constrained as the models are developed using the displacement based finite element i.e., all the unknowns are defined in terms of displacement only.

In the present work, the following sets of boundary conditions are used. However, the mathematical formulation, which is general in nature, does not put any limitations.

I. Simply supported (SSSS):

$v_0 = w_0 = \theta_y = \theta_z = \phi_y = \lambda_y = 0$ at x=0 and a;

$u_0 = w_0 = \theta_x = \theta_z = \phi_x = \lambda_x = 0$ at y=0 and b (2.33)

II. Clamped (CCCC):

$u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = \phi_x = \phi_y = \lambda_x = \lambda_y = 0$ at x=0 and a; y=0 and b; (2.34)

III. Hinged (HHHH):

$u_0 = v_0 = w_0 = \theta_z = \phi_y = \lambda_y = 0$ at x=0 and a;

$u_0 = v_0 = w_0 = \theta_z = \phi_x = \lambda_x = 0$ at y=0 and b (2.35)

IV. Free (FFFF):

$u_0 \neq v_0 \neq w_0 \neq \theta_x \neq \theta_y \neq \theta_z \neq \phi_x \neq \phi_y \neq \lambda_x \neq \lambda_y \neq 0$ at x=0 and a; y=0 and b; (2.36)

2.11 Solution Technique

The proposed models have been solved using a homemade computer code developed in MATLAB environment. The steps of solutions are discussed in detail in the following lines:

I. As a first step, the geometry and the material properties of the composite plate are defined.
II. The domain is discretised using isoparametric Lagrangian quadrilateral element.
III. The elemental stiffness matrix, mass matrix and force vectors are initialized and evaluated.
IV. The global stiffness matrix, mass matrix and force vectors are obtained by assembling the elemental matrices.
V. The boundary condition is applied to avoid rigid body motion and the respective governing equations are solved to obtain the desired responses.
VI. The static responses are computed by matrix inversion and pre-multiplication.
VII. Similarly, the free vibration responses are solved using MATLAB inbuilt symmetric eigenvalue solver by choosing the corresponding syntax. Based on the defined syntax MATLAB will solve using proper subroutine from its library (LAPACK solver).

Further, the responses are also computed using a simulation model developed in ANSYS environment with the help of APDL code for the laminated composite structure. The present ANSYS model is developed and discretised using an eight node serendipity element (SHELL281) with six degrees of freedom at each node. In this simulation model, the mid-plane kinematics of the plate is governed by the FSDT. The brief description of solution procedure in ANSYS platform are as follows:

I. As a first step, the geometry of the plate has been created.
II. A finite number of layers have been considered to achieve the laminated composite plate.
III. Each layer of the plate is assigned with necessary material properties.
IV. The created model has been discretised using SHELL281 element, from ANSYS element library, to obtain the required mesh.
V. Then, the boundary condition and the transverse load, if any are applied to obtain the required static, vibration and dynamic responses by selecting the proper syntax (linear static or Block-Lancoz).

2.12 Computational Investigations

The computer implementation of the present FEM formulation is executed through developing a suitable computer code in MATLAB R2012b environment. Different programs are built for various analyses, viz. static, free vibration and transient analysis.

The code has been developed in very general way so that it can be used to compute different types of problems of laminated composite plates. A desktop computer with the specification of Intel(R) Core(TM) i5-2400 CPU @ 3.1GHz, 4.00GB RAM, 64-bit Operating system is used to execute the MATLAB code. In addition, a 32 node high performance computing system with 64-Intel Sandy Bridge CPU @2GHz is also utilised for the purpose of result generation. Although, the present code can be done using other languages such as FORTRAN and/or C++ but MATLAB is more user friendly and it is easy to implement. Also, MATLAB takes care of the error accumulations due to the numerical analysis and selection of methodology is easier as one has to select the appropriate syntaxes from its library to get the desired output. In addition to MATLAB, simulation model is also developed using the ANSYS parametric design language (APDL) code in ANSYS 15.0 environment for the laminated composite plates which is less time consuming and accepted by many industries.

2.13 Summary

The main purpose of this present chapter is to develop the general mathematical models for the computer implementation of the proposed problem, i.e., the static and dynamic analysis of delaminated composite structures. The necessity and requirement of the problem and their background are discussed in the first section. A few indispensable assumptions taken in the analysis are pointed out in the Section 2.2. Then, the geometry of the composite laminated plate without delamination and its assumed displacement field are stated in Section 2.3. In Section 2.4, the strain-displacement relations and subsequent strain vectors were evaluated. The mathematical model for the proposed plate problem was discretised with the help of finite element methods in Section 2.5. Then, in Section 2.6 various energies and the work done were calculated. Subsequently, the geometry of the delaminated segment and its displacement field was stated in Section 2.7. Continuity conditions for the displacement were established in Section 2.8 for the laminated segment and delaminated segments which were developed independently in previous sections. Then, in Section 2.9 the governing equation of motion for the laminated plate were obtained for the static, free vibration and transient analysis of the plate. The different types of boundary conditions were presented in Section 2.10. A detailed discussion on the solution techniques was presented in Section 2.11. Finally, the computational investigations were discussed briefly in Section 2.12.

In the following chapters, the solution of governing equation for various kinds of plate problems such as static, free vibration and transient response of laminated composite plate with and without delamination have been investigated in details for different parameters and discussed.

Chapter 3

Static and Dynamic Analysis of Laminated Composite Plate

3.1 Introduction

In this chapter, the static and dynamic responses of laminated composite plates have been examined with the help of the developed mathematical models, i.e., Model-1, Model-2 and Model-3 as discussed in the former chapter. It will be appropriate to say that the laminated structural components, throughout their service life, are under the influence of various combined loading and constrained conditions. This affects the original geometry of the structure largely and the structural components are distorted which changes the entire condition in the structural analysis. Hence, the prediction of the structure's responses under loading condition is quiet inevitable to model these complex structural problems precisely with less computational effort. In Section 3.2, the governing equation of the static, free vibration and transient analysis of laminated composite plates are provided along with its solution steps. Section 3.3 presents the convergence behaviour of the proposed numerical and simulation models for the analysis of static and dynamic analysis of the laminated composite plates and then the responses were compared with the results obtained from formerly published literature and succeeding experimental studies. In continuation, the robustness and the efficacy of the presently developed models have been revealed through the comprehensive parametric study. The effects of various material parameters and the support conditions on the structural responses of the laminated composite plates are studied. Finally, this chapter is summarised with the concluding remarks in Section 3.4.

3.2 Governing Equation and Solution Methodology

The final form of equilibrium equation for static, free vibration and dynamic analysis of laminated composite plate is derived using the variational principle, Hamilton's principle and Newmark's integration, respectively as discussed in Section 2.9. Also, the detailed solution steps used in the analysis procedure are stated in Section 2.11. For the analysis, customised homemade computer codes have been developed in MATLAB environment to compute the desired responses. Furthermore, a simulation model is developed using the ANSYS parametric design language (APDL) code in ANSYS 15.0 environment for the computational purpose.

3.3 Results and Discussions

Based on the developed mathematical models as discussed in Chapter 2, FE code is developed in MATLAB environment in conjunction with finite element steps for static and dynamic analysis. A nine noded isoparametric element has been utilized with nine as well as ten degrees of freedom per node and discretised using the finite element steps. In continuation to this, a simulation model is also developed in ANSYS using ANSYS parametric design language (APDL) code to substantiate the present mathematical models. The convergence study of the static and dynamic responses of the laminated composite plate for all the developed models is executed for various geometrical and material configurations. Subsequently, in order to examine the authentication and accuracy of the present developed models, numerous examples are solved for the validation purposes and compared with those available published literature. After the comprehensive testing of the present models, an inclusive parametric study of the composite plate is performed. The effect of different combinations of parameters such as the thickness ratio (a/h), the aspect ratio (a/b), the modular ratio (E_l/E_t) and the support condition on the composite plate responses are discussed.

The static and dynamic responses of the laminated composite plate are computed for convergence and comparison purpose by considering various geometrical and material configurations as tabulated in Table 3.1.

Table 3.1: Material Properties of laminated composite plates

Properties	MATERIAL-1 (M1)	MATERIAL-2 (M2)	MATERIAL-3 (M3)	MATERIAL-4 (M4)	MATERIAL-5 (M5)	MATERIAL-6 (M6)
a	0.3048 m	0.2 m	1 m	1 m	0.25 m	0.25 m
b	0.3048 m	0.2 m	1 m	1 m	0.25 m	0.25 m
h	0.0024 m	Open	Open	a/5	0.05 m	0.01 m
E_l	12.6GPa	$25 \times E_t$	$40 \times E_t$	Open	$25 \times E_t$	$25 \times E_t$
E_t	12.63Gpa	1GPa	1GPa	1GPa	21GPa	21GPa
E_z	12.63Gpa	E_t	E_t	E_t	E_t	E_t
G_{lt}	2.155GPa	$0.5 \times E_t$	$0.6 \times E_t$	$0.5 \times E_t$	$0.5 \times E_t$	$0.5 \times E_t$
G_{tz}	2.155GPa	$0.5 \times E_t$	$0.5 \times E_t$	$0.35 \times E_t$	$0.5 \times E_t$	$0.5 \times E_t$
G_{lz}	2.155GPa	$0.2 \times E_t$	$0.6 \times E_t$	$0.5 \times E_t$	$0.5 \times E_t$	$0.2 \times E_t$
υ_{lt}	0.023949	0.25	0.25	0.3	0.25	0.25
υ_{lz}	0.023949	0.25	0.25	0.3	0.25	0.25
υ_{tz}	0.023949	0.25	0.25	0.3	0.25	0.25
ρ	1 Kgm^{-3}	1 Kgm^{-3}	1 Kgm^{-3}	1 Kgm^{-3}	800 Kg/m^3	800 Kg/m^3

3.4 Convergence and Validation Study of Static Response

In order to check the consistency and the stability of the proposed FE models, the static responses of the laminated composite plate are computed for different mesh refinement. The convergence study of the static responses of the plate is performed by employing the higher order models (Model-1 and Model-2) and the simulation model (Model-3). Successively, the validity of the present numerical models has been checked by comparing the responses with those available published literature.

3.4.1 Convergence behaviour of central deflection of laminated composite plate

As a first step, the convergence behaviour of the central deflection of the developed models is presented in Figure 3.1. For the computation purpose, a four layer clamped symmetric cross-ply square plate is considered under uniformly distributed load (UDL) by taking the same material and geometrical parameters as in Zhang and Kim [85] and tabulated as M1 in Table 3.1. It can be seen that the results are converging well with the mesh refinement for all the developed models and a (6×6) mesh is sufficient to compute the static response of the plate. So, based on the convergence study, a (6×6) mesh has been used to compute the new results throughout the analysis.

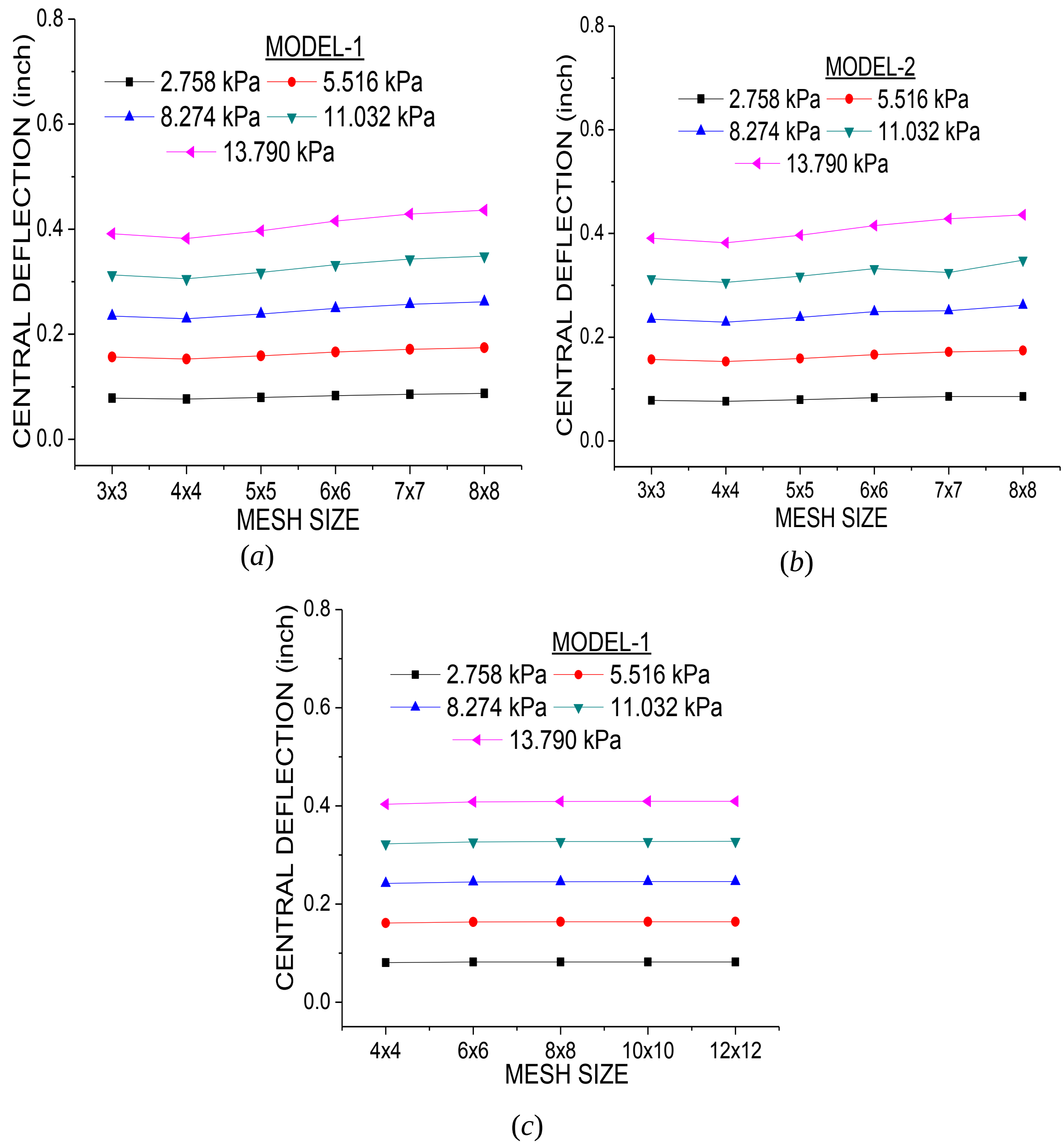


Figure 3.1(*a*)-(*c*) Convergence of central deflection (mm) of a clamped four layered cross-ply symmetric laminated composite plate subjected to UDL

3.4.2 Comparison study of central deflection of laminated composite plate

In this example, the central deflection of four layer clamped symmetric cross-ply square laminated composite plate subjected to UDL is computed using the proposed models and compared simultaneously with the previous published literature (Zhang and Kim [85] and Putcha and Reddy [86]) and presented in Table 3.2. The geometrical and material parameters are taken to be same as the reference. It is clear from the table that the present numerical results are showing good agreement with those of the references.

Table 3.2**: Central deflection (mm) of clamped four layered cross-ply symmetric laminated composite plate subjected to UDL**

Load (kPa)	Central Deflection (mm)					
	Model-1	Model- 2	Model-3	Zhang and Kim[85]	Putcha and Reddy [86]	Experiment [85]
2.758	2.1107	2.1107	2.0795	2.1387	2.2098	2.2098
5.516	4.2215	4.2189	4.1580	4.27734	4.191	4.6482
8.274	6.3348	6.3297	6.2382	6.4135	6.1976	6.9596
11.032	8.4455	8.2474	8.3160	8.5522	8.382	8.9916
13.79	10.5562	10.5486	10.3962	10.6908	10.3886	11.2776

3.4.3 Convergence study of nondimensional central deflection of laminated composite plate

The convergence studies of the nondimensional static deflection of the laminated composite plate have been computed using the developed models, Model-1, Model-2 and Model-3 and presented in Figure 3.2 (a)-(e), respectively. The nondimensional static deflections are computed for different layered square simply supported cross-ply laminated composite plates under UDL. The nondimensional central deflections are obtained for three different thickness ratios (h/a = 0.05, 0.1 and 0.2) using the same material and geometrical parameters as in Xiao *et al.* [87] and listed as M2 in Table 3.1. The nondimensionalisation of the central deflection is computed using the formulae: $\bar{w} = \left(100 E_t h^3 / q_0 a^4\right) w$, where w is the maximum central deflection. It is clearly observed that the present results are converging well and a (6×6) mesh has been used for further computation.

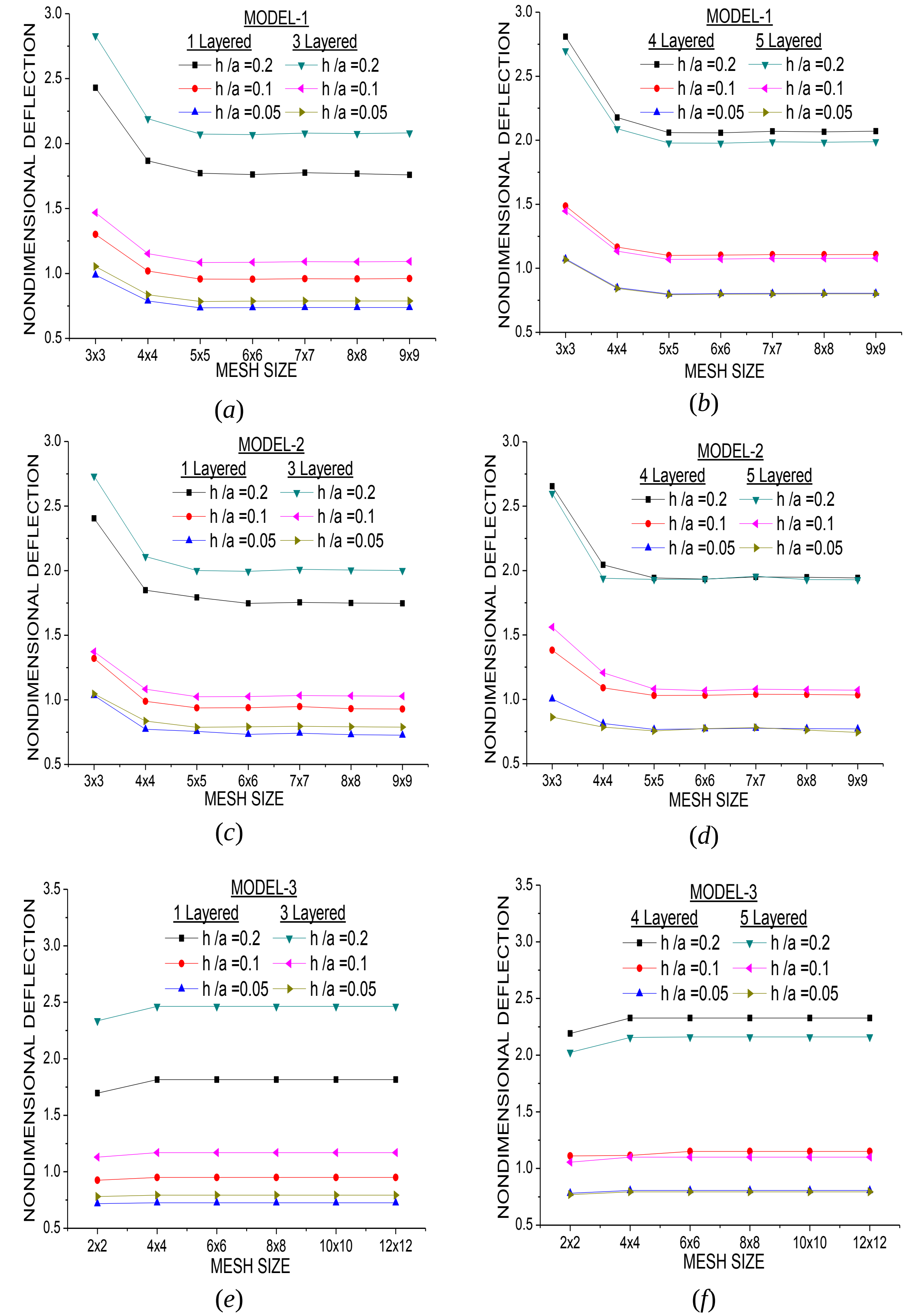


Figure 3.2 (*a*)-(*f*): Convergence of nondimensional deflection of simply supported cross-ply laminated composite plates subjected to UDL

3.4.4 Comparison study of nondimensional central deflection of laminated composite plate

Table 3.3 tabulates the nondimensional central deflection ($\bar{w}$) of simply supported cross-ply laminated composite plates subjected to UDL for different number of layers. Model-1, Model-2 and Model-3 have been employed to calculate the responses and simultaneously the results have been compared with the previous published literature (Xiao *et al.* [87], Reddy [81] and Belinha and Dinis [88]). The geometrical and material parameters are taken to be same as the reference. It is quite evident that the present numerical results are showing good agreement with those of the references.

Table 3.3: Nondimensional deflection of simply supported cross-ply laminated composite plates subjected to UDL

No. of Layers	*h*/*a*	Nondimensional Deflection ($\bar{w}$)							
		Model -1	Model -2	Model -3	3D FEM Xiao *et al.* [87]	HSDT MQ Xiao *et al.* [87]	HSDT TPS Xiao *et al.* [87]	Reddy [81]	Belinha and Dinis [88]
1	0.2	1.761	1.7468	1.816	1.7783	1.7696	1.7624	-	-
	0.1	0.956	0.9395	0.95	0.9478	0.94	0.935	0.9519	0.9537
	0.05	0.7367	0.7334	0.725	0.7255	0.7169	0.7094	0.7262	0.7281
3	0.2	2.069	1.9344	2.464	2.3218	2.1644	2.1556	-	-
	0.1	1.0863	1.032	1.17	1.1541	1.086	1.08	1.0219	1.0225
	0.05	0.7866	0.771	0.7938	0.7951	0.7688	0.7613	0.7572	0.7583
4	0.2	2.0582	1.995	2.328	2.2383	2.1496	2.14	-	-
	0.1	1.1027	1.026	1.15	1.1401	1.109	1.0955	1.025	1.0248
	0.05	0.8024	0.7912	0.8063	0.8029	0.7844	0.7775	0.7694	0.7698
5	0.2	1.9773	1.9322	2.16	2.1044	1.81	1.8032	-	-

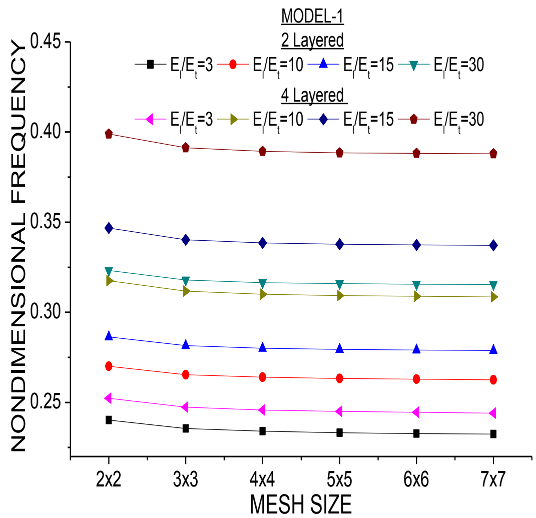
	0.1	1.073	1.068	1.1	1.0576	0.973	0.9675	0.9727	0.9722
	0.05	0.7973	0.773	0.7938	0.7794	0.7506	0.7438	0.7581	0.7584

3.5 Convergence and Validation Study of Dynamic Analysis

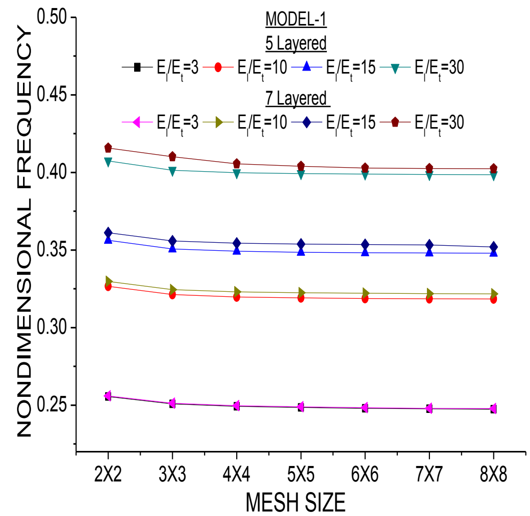
Now, in order to depict the applicability of the proposed numerical and simulation models, the models are extended to show the free vibration response and transient response of the laminated composite plate. The convergence behavior of the free vibration responses for all the developed models, Model-1, Model-2 and Model-3 has been studied for different mesh refinement and then validated by comparing the responses with those available published literature. Similarly, the transient responses of the laminated composite plates are computed using the proposed models which are then compared with the transient responses available in published literature.

3.5.1 Convergence behaviour of nondimensional natural frequency of laminated composite plate

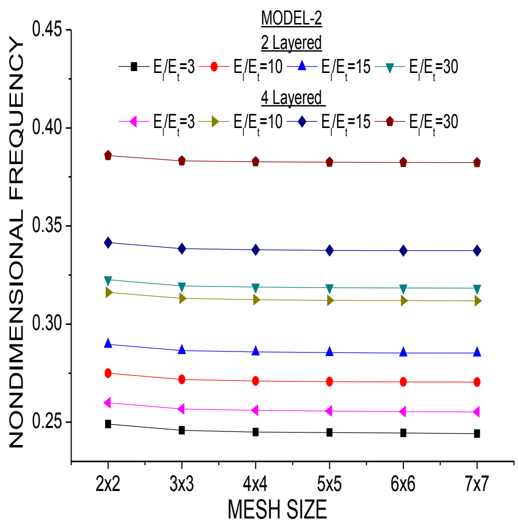
The convergence behaviour of the nondimensional natural frequency has been studied by employing the developed models (Model-1, Model-2 and Model-3). Figure 3.3 (a)-(f) presents the free vibration responses for a two and four layered simply supported square cross-ply laminated composite plate by taking the same material and geometrical parameters as in Kant and Swaminathan [29] and tabulated as M3 material properties in Table 3.1. The nondimensionalisation of the natural frequency is calculated using the formula: $\overline{\omega} = \left(\omega b^2/h\right)\sqrt{\rho/E_t}$, where ω is the natural frequency of the laminated plate. From the figure, it can be noted that for all the developed models the results are converging well with the mesh refinement and (6×6) mesh is sufficient to compute the free vibration response of the laminated plate further.


MODEL-1
2 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
4 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
NONDIMENSIONAL FREQUENCY
0.45
0.40
0.35
0.30
0.25
2x2
3x3
4x4
5x5
6x6
7x7
MESH SIZE

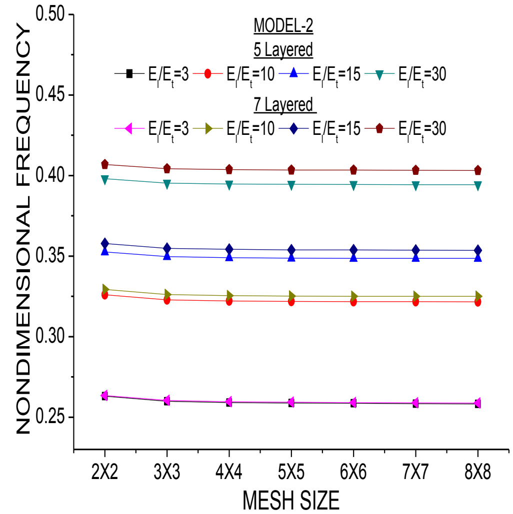
(a)


MODEL-1
5 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
7 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
NONDIMENSIONAL FREQUENCY
0.50
0.45
0.40
0.35
0.30
0.25
2X2
3X3
4X4
5X5
6X6
7X7
8X8
MESH SIZE

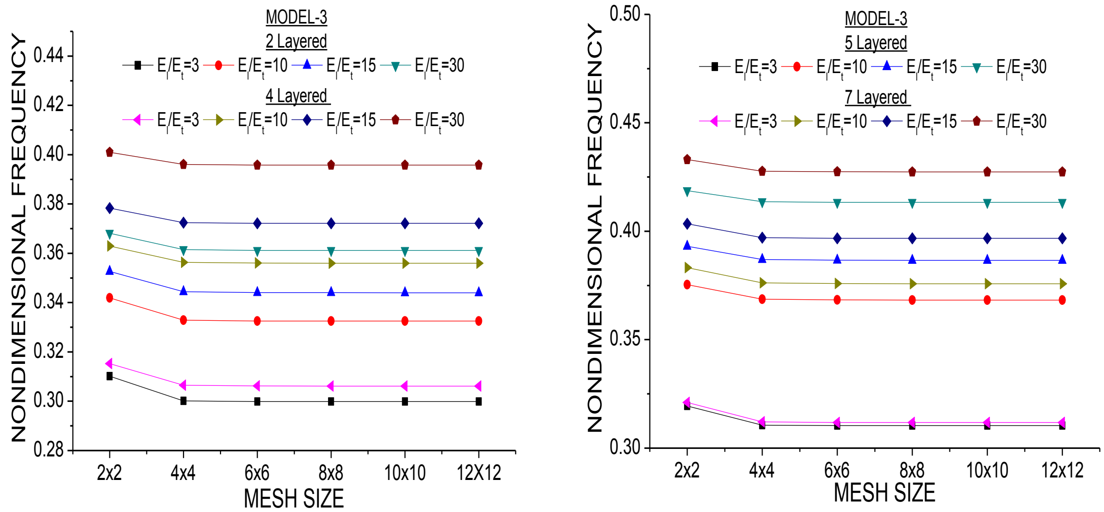
(b)


MODEL-2
2 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
4 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
NONDIMENSIONAL FREQUENCY
0.45
0.40
0.35
0.30
0.25
2x2
3x3
4x4
5x5
6x6
7x7
MESH SIZE

(c)


MODEL-2
5 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
7 Layered
E/E_t=3
E/E_t=10
E/E_t=15
E/E_t=30
NONDIMENSIONAL FREQUENCY
0.50
0.45
0.40
0.35
0.30
0.25
2X2
3X3
4X4
5X5
6X6
7X7
8X8
MESH SIZE

(d)



(*e*) (*f*)

Figure 3.3 (*a*)-(*f*): Convergence of nondimensional frequency of two and four layered simply supported cross-ply laminated plate

3.5.2 Comparison study of nondimensional natural frequency of laminated composite plate

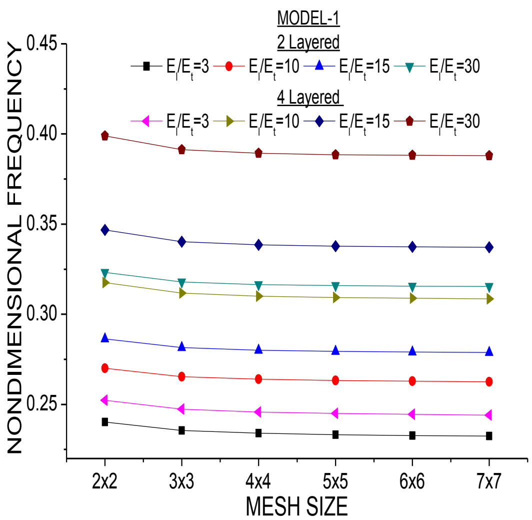
Now, the previous problem has been extended for the validation purpose for all the three models (Model-1, Model-2 and Model-3). In this example, the nondimensional natural frequency ($\bar{\omega}$) of a two and four layered simply supported square cross-ply laminated composite plate have been computed and these responses are consecutively compared with the previous published literature (Kant and Swaminathan [29], Whitney and Pagano [89], Reddy [90] and Senthilnathan et al. [91]) and presented in Table 3.4. The geometrical and material parameters are taken to be same as the reference. It is clear from the table that the present numerical results are showing decent agreement with those of the references. In addition, a considerable amount of differences between the results is also seen for the thick laminates (a/h = 2 and 4) because of the nonzero values of shear stresses at the top and bottom of the plate to avoid more mathematical assumptions.

Table 3.4: Nondimensional frequency of two and four layered simply supported cross-ply laminated plate

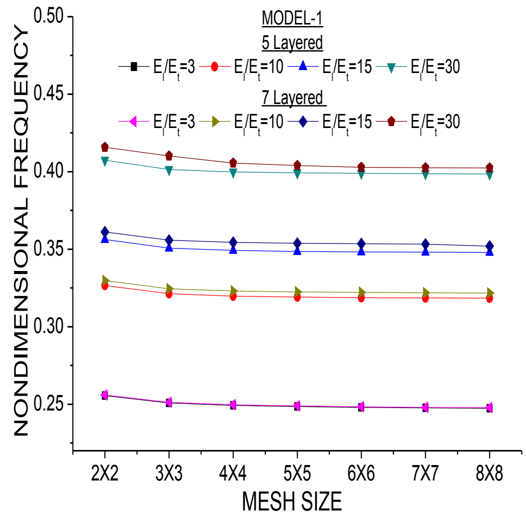
No. of layers	*a*/*h*	Nondimensional Frequency ($\bar{\omega}$)						
		Model-1	Model-2	Model-3	Kant and Swaminathan [29]	Whitney and Pagano [89]	Reddy [90]	Senthilnathan *et al.* [91]
2	2	0.2327	0.2445	0.2999	0.2392	0.2388	0.2389	0.2327
	4	0.2629	0.2705	0.3325	0.2671	0.2675	0.2669	0.2629
	10	0.2791	0.2853	0.2853	0.2815	0.2809	0.2812	0.2791
	20	0.3156	0.3184	0.3611	0.3117	0.3117	0.3116	0.3156
	50	0.3336	0.3396	0.3667	0.3256	0.3236	0.3255	0.3336
	100	0.2445	0.2554	0.3062	0.2493	0.2495	0.2491	0.2445

3.5.3 Convergence behaviour of nondimensional natural frequency of laminated composite plate

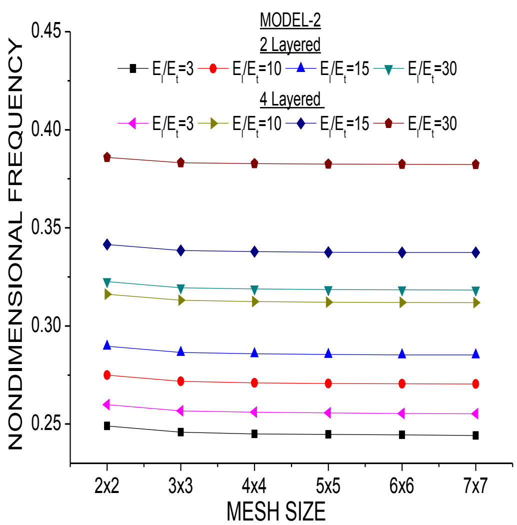
All the developed models, Model-1, Model-2 and Model-3 are engaged to study the convergence behaviour of the free vibration of square simply supported cross-ply laminated composite plate comprising of different number of layers (2, 4, 5 and 7). The nondimensional frequency of the plate are plotted in Figure 3.4 (a)-(f) by taking the same material and geometrical parameters as M4 material properties of Table 3.1, as in Matsunaga [31]. The nondimensionalisation of the natural frequency is calculated using the formulae: $\hat{\omega} = \omega h\sqrt{\rho/E_t}$, where ω is the natural frequency of the laminated plate. It can be noted that for all the developed models the results are converging well with the mesh refinement and (6×6) mesh is sufficient to compute the free vibration response of the laminated plate. Therefore, throughout the analysis, a (6×6) mesh has been used to compute the new results based on the convergence study.


MODEL-1
2 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
4 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.25 0.30 0.35 0.40 0.45
2x2 3x3 4x4 5x5 6x6 7x7
MESH SIZE

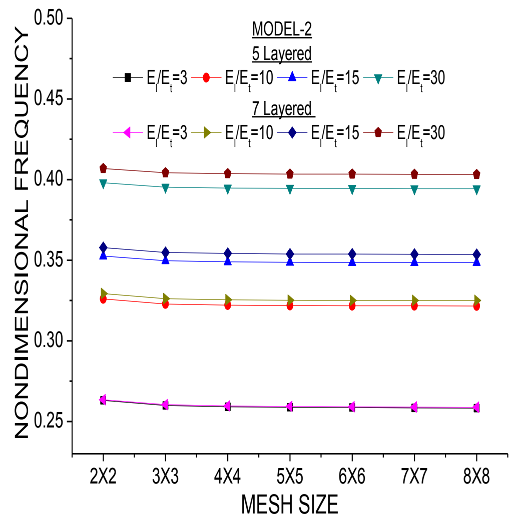
(*a*)


MODEL-1
5 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
7 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.25 0.30 0.35 0.40 0.45 0.50
2X2 3X3 4X4 5X5 6X6 7X7 8X8
MESH SIZE

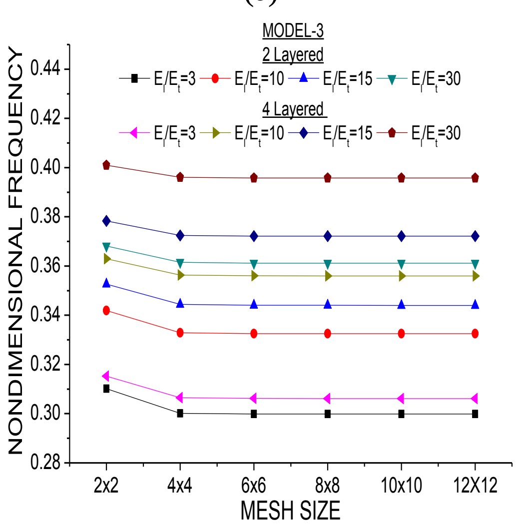
(*b*)


MODEL-2
2 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
4 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.25 0.30 0.35 0.40 0.45
2x2 3x3 4x4 5x5 6x6 7x7
MESH SIZE

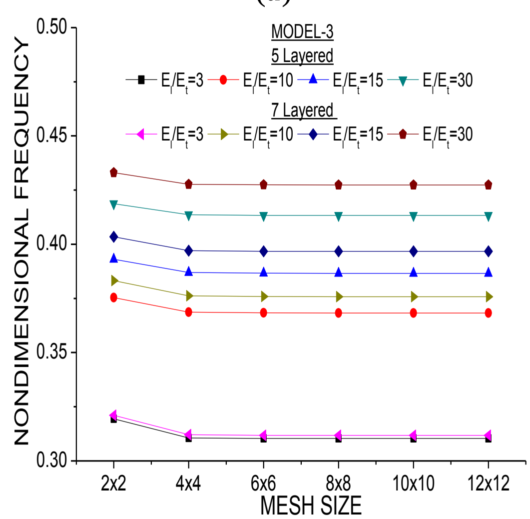
(*c*)


MODEL-2
5 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
7 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.25 0.30 0.35 0.40 0.45 0.50
2X2 3X3 4X4 5X5 6X6 7X7 8X8
MESH SIZE

(*d*)


MODEL-3
2 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
4 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.28 0.30 0.32 0.34 0.36 0.38 0.40 0.42 0.44
2x2 4x4 6x6 8x8 10x10 12X12
MESH SIZE

(*e*)


MODEL-3
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El/Et=3 El/Et=10 El/Et=15 El/Et=30
7 Layered
El/Et=3 El/Et=10 El/Et=15 El/Et=30
NONDIMENSIONAL FREQUENCY
0.30 0.35 0.40 0.45 0.50
2x2 4x4 6x6 8x8 10x10 12x12
MESH SIZE

(*f*)

Figure 3.4 (*a*)-(*f*): Convergence of nondimensional frequency of square simply supported cross-ply laminated composite plate

3.5.4 Comparison study of nondimensional natural frequency of laminated composite plate

The previous problem is now being extended for the validation purpose by comparing the present results with the previous published literature (Noor and Burton [92], Kant and Kommineni [93] and Matsunaga [31]) and presented in Table 3.5. The geometrical and material parameters are taken to be same as the reference. It is clear that the present numerical results are showing decent agreement with those of the references.

Table 3.5: Nondimensional frequency of simply supported cross-ply laminated plate

No. of layers	E_l/E_t	Nondimensional Frequency ($\hat{\omega}$)					
		Model-1	Model-2	Model-3	Noor and Burta [92]	Kant and Kommineni [93]	Matsunaga [31]
2	3	0.2327	0.2445	0.2999	0.2392	0.2388	0.2389
	10	0.2629	0.2705	0.3325	0.2671	0.2675	0.2669
	15	0.2791	0.2853	0.2853	0.2815	0.2809	0.2812
	30	0.3156	0.3184	0.3611	0.3117	0.3117	0.3116
	40	0.3336	0.3396	0.3667	0.3256	0.3236	0.3255
4	3	0.2445	0.2554	0.3062	0.2493	0.2495	0.2491
	10	0.3089	0.312	0.356	0.3063	0.3002	0.3063
	15	0.3374	0.3375	0.372	0.3307	0.3306	0.3309
	30	0.3882	0.3824	0.3958	0.3726	0.3725	0.3731
	40	0.4081	.3996	0.404	0.3887	0.3899	0.3893
5	3	0.248	0.2586	0.3103	0.2527	0.2528	0.2529
	10	0.3188	0.3218	0.3683	0.3173	0.3201	0.3195
	15	0.3483	0.3486	0.3866	0.3437	0.347	0.347
	30	0.399	0.3945	0.4133	0.3876	0.3935	0.3931
	40	0.4186	0.412	0.4225	0.404	0.4121	0.4102
7	3	0.2484	0.259	0.3103	0.2535	0.2534	0.2533
	10	0.3221	0.3251	0.3683	0.3218	0.3224	0.3222
	15	0.3535	0.3538	0.3866	0.3505	0.352	0.3514
	30	0.4028	0.4034	0.4133	0.399	0.4004	0.4005
	40	0.4212	0.4224	0.4225	0.4173	0.4204	0.419

3.5.5 Comparison study of nondimensional transient response of laminated composite plate

In this example, models are applied to compute the transient responses of two and eight layered cross-ply square simply supported composite plate. The geometrical parameters and material properties used for this analysis are tabulated in Table 3.1 as M6 material properties. The transient responses are computed under uniform step load, by setting the time step as 5μs. Figure 3.6 (a) and (b) present the nondimensional deflections computed by employing the developed models (Model-1, Model-2 and Model-3) for two and eight layered laminated composite plates, respectively and subsequently compared with the available published results (Chen and Dawe [94] and Wang et al. [95]). It can be clearly observed that the results are showing very good agreement with the references.

3.5.6 Comparison study of nondimensional transient response of laminated composite plate

The proposed models are applied to compute the transient responses of two and eight layered cross-ply square simply supported composite plate. The geometrical parameters and material properties used for this analysis are tabulated in Table 3.1 as M6 material properties. The transient responses are computed under uniform step load, q_0 by setting the time step as 5μs. Figure 3.6 (a) and (b) present the nondimensional deflections ($\bar{w}$) computed by employing the developed models (Model-1, Model-2 and Model-3) for two and eight layered laminated composite plates, respectively and subsequently compared with the available published results (Maleki et al. [39] and Reddy [81]). It can be clearly observed that the results are showing very good agreement with the references.

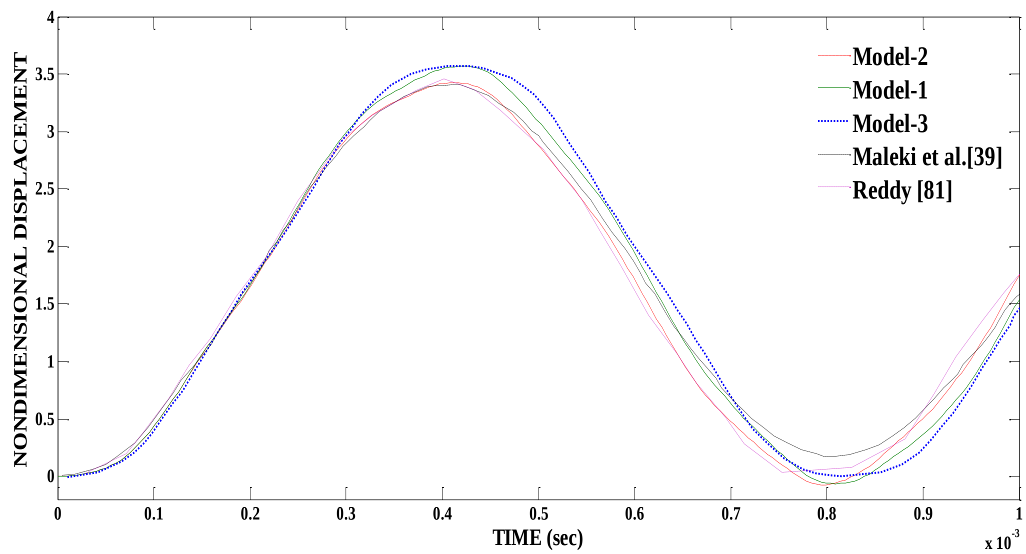


Figure 3.6 (a): Nondimensional deflection of a simply supported two layered cross-ply laminated composite plate

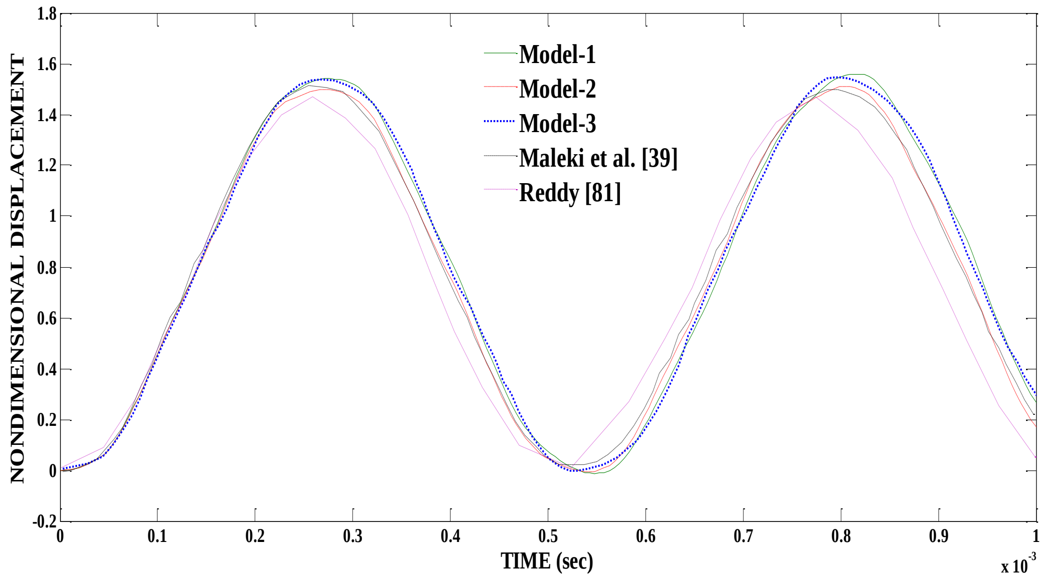


Figure 3.6 (*b*): Nondimensional deflection of a simply supported eight layered cross-ply laminated composite plate

3.6 Experimental Studies

Now, in order to build more confidence on the presently developed numerical models, static responses of different laminated plates are examined experimentally and subsequently compared with those of the developed models. The static responses of specimens from woven Glass/Epoxy, Glass fibre reinforced polymer (GFRP) and Carbon fibre reinforced polymer (GFRP) composite plates with different lamination schemes are obtained experimentally and these responses are compared with the responses obtained numerically employing the proposed models.

3.6.1 Material property evaluation

The material properties of these specimens have to be also evaluated experimentally so that these values can used in the numerical models in order to calculate the static and free vibration responses. The Young's modulus of the specimen is obtained via a unidirectional tensile test through the universal testing machine (UTM) INSTRON 1195 at National Institute of Technology (NIT), Rourkela. Three pieces are cut from each of the specimens in the longitudinal, the transverse and at an inclined angle (angle of inclination 45° to the longitudinal direction) to compute the Young's modulus in the respective directions. The specimen dimensions are prepared in accordance to ASTM standard (D 3039/D 3039M). The properties of the specimens are obtained in the said UTM by setting the loading rate as 1 mm/minute. The UTM and the deformed specimen configurations are shown in Figure 3.7(a) and (b), respectively.

Figure 3.7 (*a*): UTM INSTRON 1195

Figure 3.7 (*b*): Laminated composite specimens after tensile test

The material properties of various specimens hence found out are presented in Table 3.6. It is important to mention that Poisson's ratio for the present computational purpose is taken to be 0.17 as in Crawley [96]. Similarly, the shear modulus of the individual specimen has been computed based on the data obtained from the experiment and the given formula as in Jones [97]: $\left(G_{lt} = 1 \Big/ \left(\frac{4}{E_{45}} - \frac{1}{E_l} - \frac{1}{E_t} - \frac{2\upsilon_{12}}{E_l} \right) \right)$

Table 3.6: Material Properties of laminated composite plates

Properties	MATERIAL-7 (M7)	MATERIAL-8 (M8)	MATERIAL-9 (M9)	MATERIAL-10 (M10)	MATERIAL-11 (M11)	MATERIAL-12 (M12)	MATERIAL-13 (M13)
Material	Woven Glass/Epoxy	GFRP	GFRP	GFRP	CFRP	CFRP	CFRP
Lamination scheme	$(0^0/90^0)_5$	$(\pm 45^0)$	$(\pm 45^0)_2$	$(0^0/90^0)_2$	$(\pm 45^0)$	$(\pm 45^0)_2$	$(0^0/90^0)_2$
E_l (GPa)	5.802	4.669	4.408	5.639	6.695	6.469	10.45
E_t (GPa)	4.966	4.351	4.081	4.926	6.314	5.626	12.34
E_z (GPa)	4.966	4.351	4.081	4.926	6.314	5.626	10.45

G_{lt} (GPa)	1.898	3.25	1.1	0.75	2.7	2.05	6.45
G_{tz} (GPa)	0.949	3.25	1.1	0.75	1.35	1.025	3.225
G_{lz} (GPa)	1.898	1.625	0.55	0.375	2.7	2.05	6.45
υ_{lt}	0.17	0.17	0.17	0.17	0.3	0.3	0.3
υ_{lz}	0.17	0.17	0.17	0.17	0.3	0.3	0.3
υ_{tz}	0.17	0.17	0.17	0.17	0.3	0.3	0.3
ρ (Kgm^{-3})	1646	1900	1900	1900	1388	1388	1388

3.6.2 Static response

The static responses are computed via a Three-point bend test on a Universal Testing Machine (UTM), INSTRON 5967 with 30 kN load cell at National Institute of Technology, Rourkela. The specimens for the experimental analysis have been prepared as per ASTM standard (D 3039/D 3039M). The recommended loading rate for the analysis is fixed as 2 mm/minute. The three-point bend test setup and some of the specimens are presented in Figure 3.8 (a) and (b), respectively.

Figure 3.8 (*a*): Three point bend test set up (INSTRON 5967)

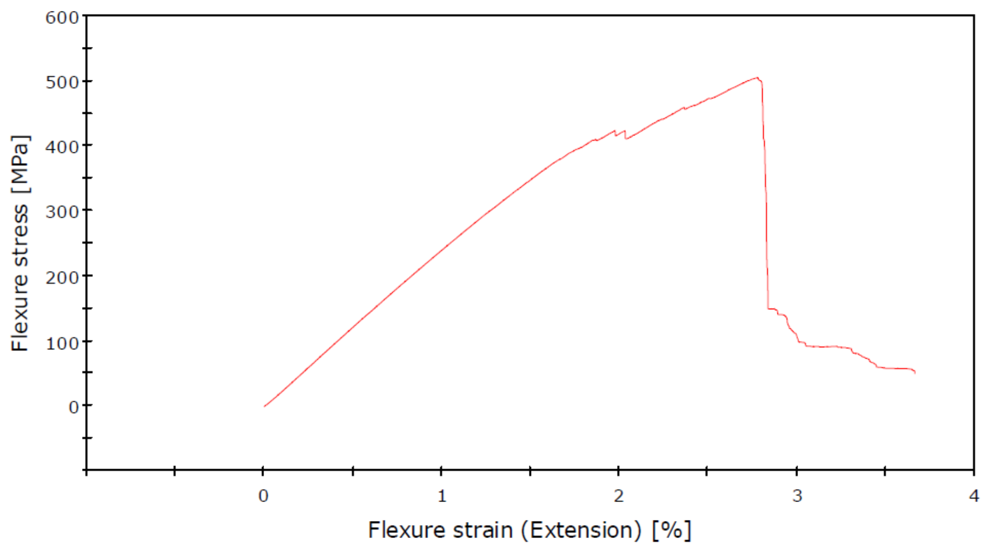
Figure 3.8 (*b*): Deformed laminated composite specimens after three point bend test

The experimental results are further compared with all the numerical and simulation results i.e., Model 1, Model 2 and Model 3 and presented in Table 3.7. It is interesting to note that both the HSDT models are showing very good agreement with those to the experimental results and it also indicates the necessity of the present HSDT type model for the analysis of laminated structures.

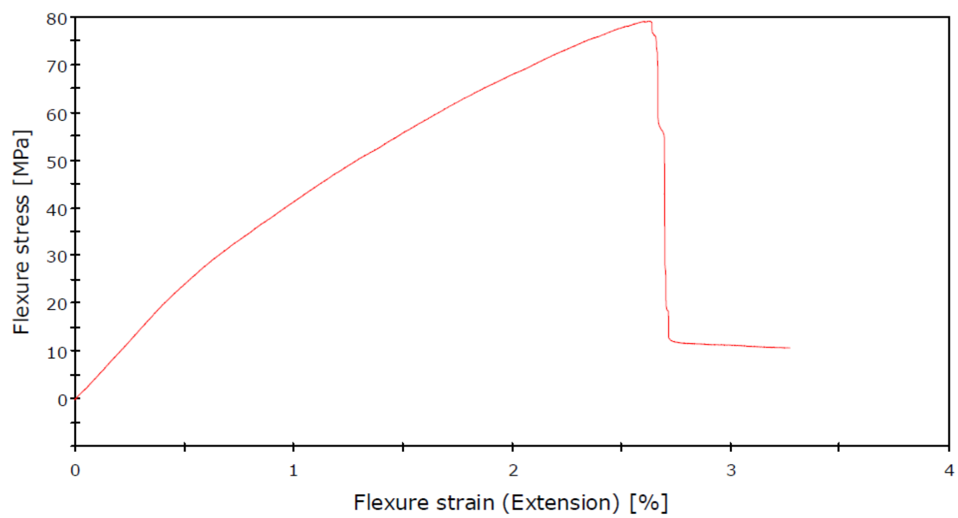
Table 3.7: Central deflection (mm) of laminated composite plate subjected to different point loads

Material	Lamination Scheme	Load (N)	Central Deflection (mm)			
			Experimental Results	Model-1	Model-2	Model-3
Woven Glass/Epoxy	$(0^0/90^0)_5$	30.003	0.0801	0.0737	0.0696	0.0706
		50.001	0.1268	0.1227	0.116	0.118
		75.037	0.1824	0.1842	0.1741	0.176
		100.090	0.2391	0.2457	0.2323	0.235
		125.036	0.2957	0.3069	0.2901	0.294

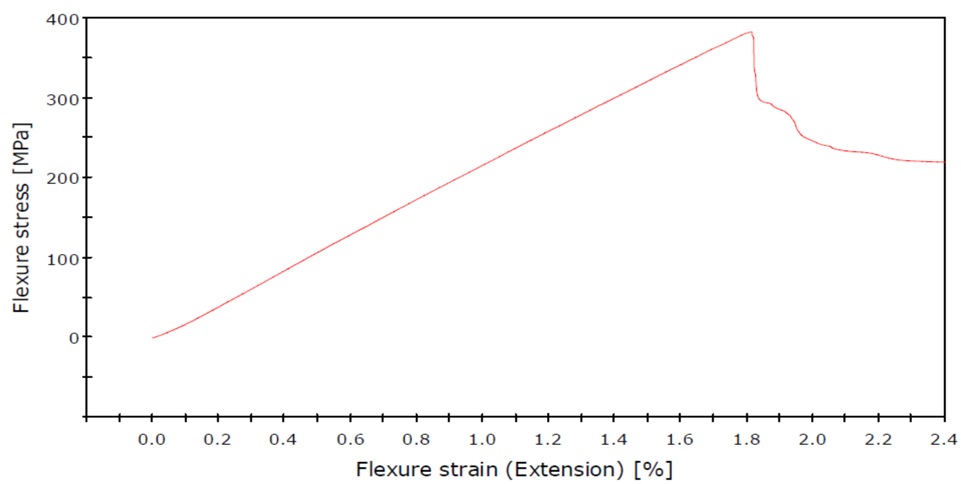
In addition, the three point bend test is also provided the flexural stress-strain diagram of the laminated composite plates. Figure 3.9 (a)-(c) presents the stress-strain diagram of ten layered woven Glass/Epoxy, four layered GFRP and CFRP cross-ply laminated composite plates during the three-point bend test, respectively.



(*a*)



(*b*)



(*c*)

Figure 3.9 (*a*)-(*c*): Stress-strain diagram of laminated composite plates during the three-point bend test

3.6.3 Free vibration response

The experimental study is also performed for the free vibration analysis using the same composite laminate specimens as taken in the static analysis. The natural frequency of various composite plates are recorded experimentally through the PXIe-1071 (National Instruments) at NIT Rourkela. Figure 3.10(a) depicts the various components for the free vibration experimental set-up. The vibration responses of the composite plate (3) under CFFF support condition, with the help of a fixture (5) are recorded via an accelerometer (4) that is mounted on the centre of the plate. The plate is excited with the help of an impact hammer (6) on any random points and through the accelerometer, the signal is captured. The accelerometer is a type of sensor that sense the vibration and convert it into the analogue voltage signal. These analogue voltage signal are received through the PXIe (1) which is an eight-slot chassis that features a high-bandwidth backplane to meet a wide range of high-performance test and measurement application needs which is ideal for processor-intensive, deterministic modular instrumentation and data acquisition real-time applications. An inbuilt analogue to digital converter within the PXIe converts the signals back to digital form which is then processed with the help of LABVIEW software. In LABVIEW, a virtual instrument (VI) program circuit is developed which primarily comprises of three components: a back panel, a front panel and a connector panel. These three component helps in supply of the input and show the output on the computer screen (2). Figure 3.10(b) depicts the block diagram of the LABVIEW software for the data acquisition of the desired signal and subsequent analysis. The velocity and displacement of the plate can be evaluated from the recorded acceleration as shown in the LABVIEW circuit using the single and double integration blocks. In addition, the acceleration signals are passed through the power spectrum module of the LABVIEW for fast Fourier transformation to obtain the frequency domain responses. The peaks of the frequency response spectrum give the natural frequencies of vibration for different modes. The peaks of the frequency response spectrum give the natural frequencies of vibration for different modes. The first peak of the frequency response for four layered angle ply GFRP and CFRP plates are shown in Figure 3.10(c)-(d) which gives the first natural frequency of the respective plates.

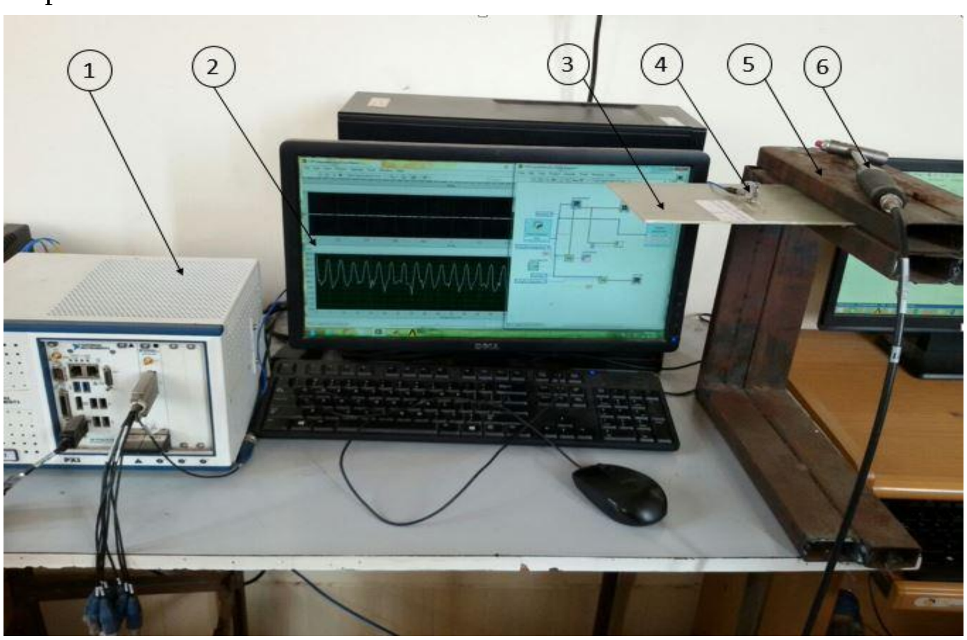


Figure 3.10 (*a*): Experimental set up 1.NI PXIe 1071 2.Computer screen 3.Laminated composite plate 4.Accelorometer 5.Fixture 6.Impact hammer

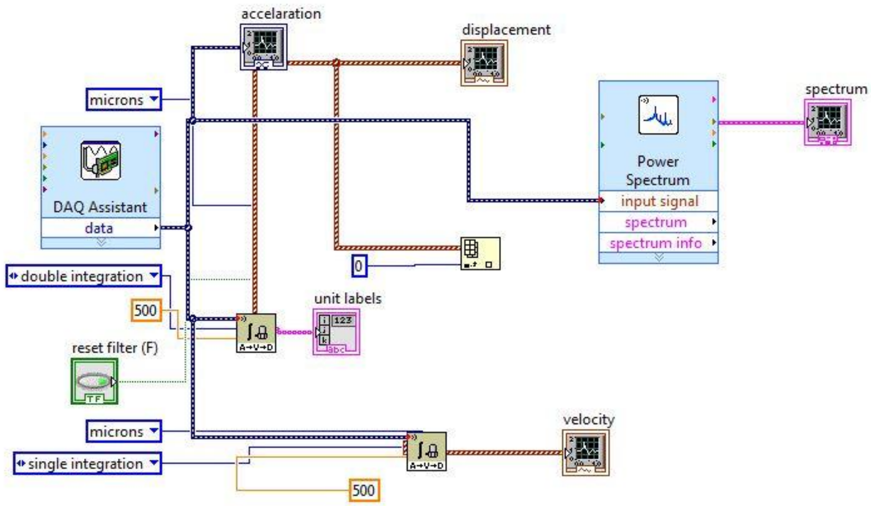


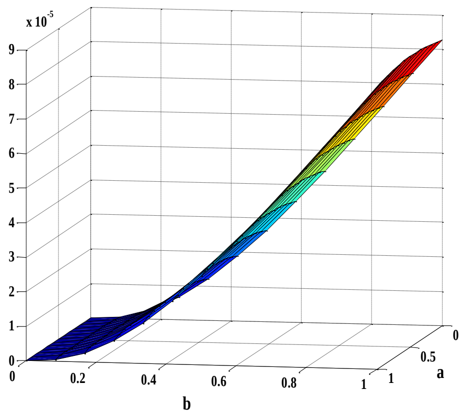
Figure 3.10 (*d*): First peak of frequency response of four layered angle ply CFRP plate

Finally, the experimental results are compared with those numerical results based on the FE models and presented in Table 3.8. The comparison study clearly indicates the necessity of present HSDT models (Model-1 and Model-2) instead of FSDT (Model-3) for the analysis of laminated structures.

Table 3.8: Natural frequency of cantilever laminated composite plates

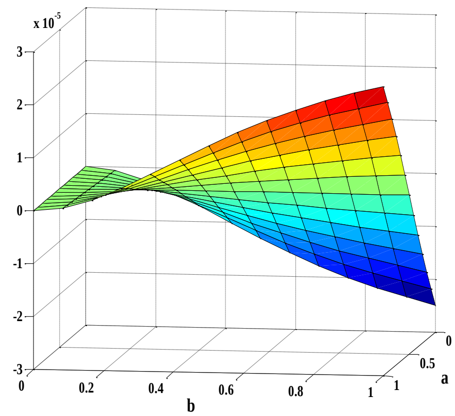
Material	Lamination Scheme	Mode No.	Natural Frequency (Hz)			
			Experimental Results	Model-1	Model-2	Model-3
Woven Glass/Epoxy	$(0^0/90^0)_5$	1	68.213	70.218	70.113	65.404
		2	177.5	164.295	162.91	161.589
		3	432.5	440.461	435.973	431.44
		4	533.5	551.317	516.776	563.37
		5	601.5	608.928	603.270	596.00

In addition, the mode shapes of the respective natural frequencies of the laminated plates are also plotted by employing one of the higher order models, i.e., Model-1. It is true that the mode shapes show the direction of vibration only but not give any idea related to the numerical value. Figure 3.11(a)-(e) presents the mode shapes of the first five natural frequencies of cantilever four layered symmetric angle-ply GFRP laminated plate. It can be clearly seen that the present responses are within the expected line.



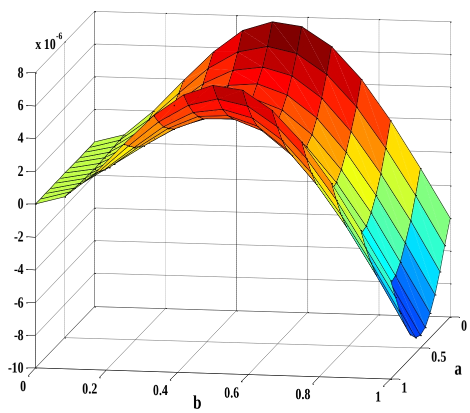
Natural frequency=39.00986

(*a*)



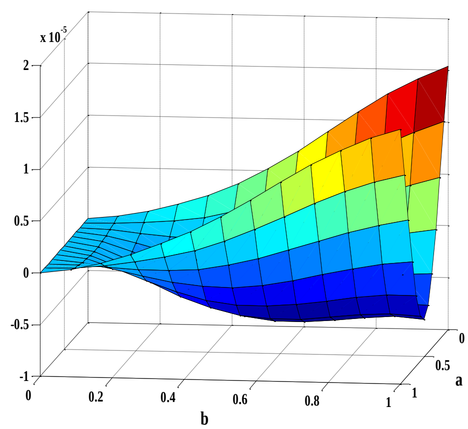
Natural frequency=109.4509

(*b*)



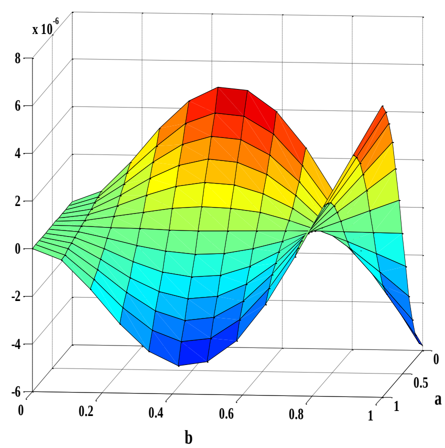
Natural frequency=240.5947

(*c*)



Natural frequency=324.0557

(*d*)



Natural frequency=383.6592

(*e*)

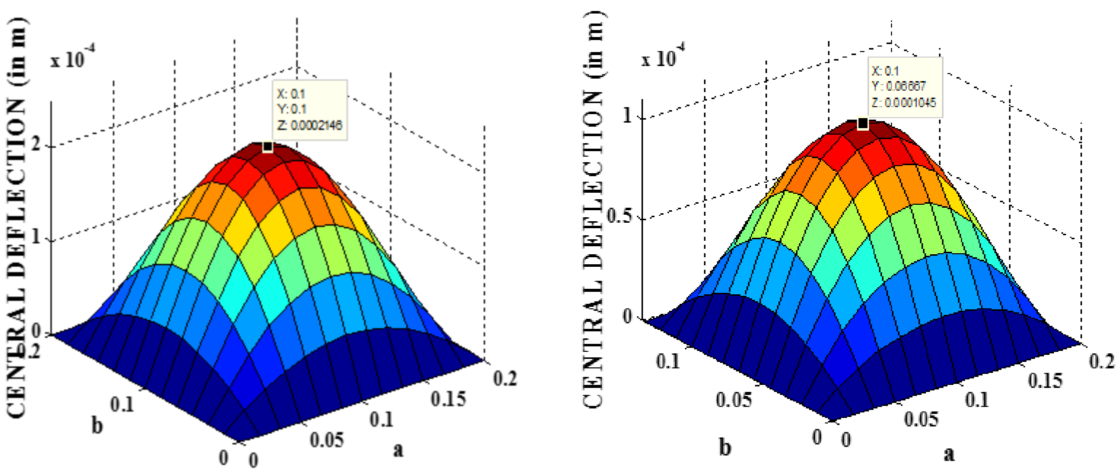
Figure 3.11 (*a*)-(*e*): Mode shapes of first five natural frequencies of cantilever four layered symmetric angle-ply Glass/Epoxy laminated plate

3.7 Numerical Illustrations

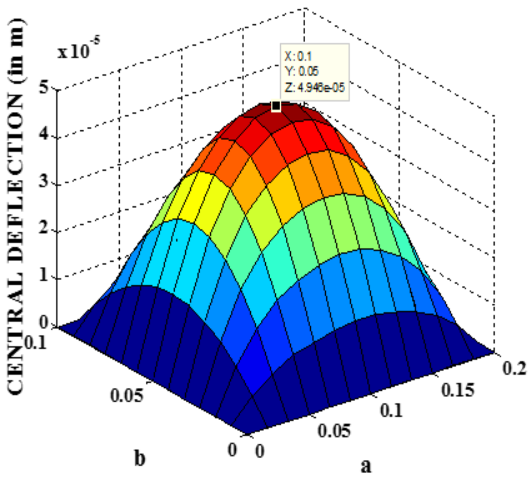
Based on the outcomes of the earlier sections, it can be clearly concluded that the present higher-order mathematical models are essential and inevitable for the accurate analysis of laminated composite plates. In continuation of the above section, the robustness of the presently developed finite element models are demonstrated through a variety of numerical problems considering different material and geometrical parameters. The static, free vibration and transient responses are computed for different geometrical (the thickness ratios and the aspect ratios), material (the modular ratios) and the support conditions and discussed in detailed in the following subsections.

3.7.1 Effect of aspect ratio on static response of laminated composited plate

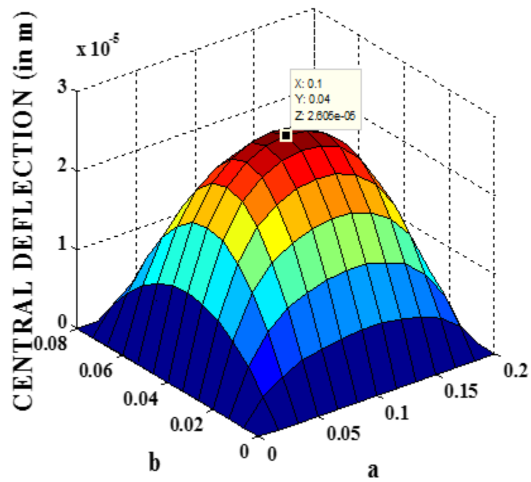
The aspect ratio (a/b) is a vital feature to determine the stiffness and the stability of any structural element, mainly for the case of laminated structures. The central deflection (w) of five layered simply supported cross-ply symmetric laminated composite plate subjected to UDL are analyzed by employing Model-1. The deformation shape of the laminated plate with a/h=10 and M2 material properties as in Table 3.1 are plotted for different aspect ratio (a/b=1, 1.5, 2, 2.5 and 3) and presented in Figure 3.12. It is apparent from the deformation shapes that as the aspect ratio increases, the central deflection tends to decrease.



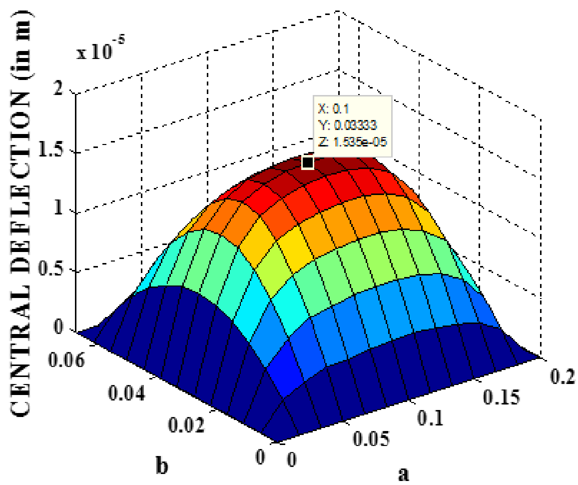
(a) (*b*)



(*c*)



(*d*)

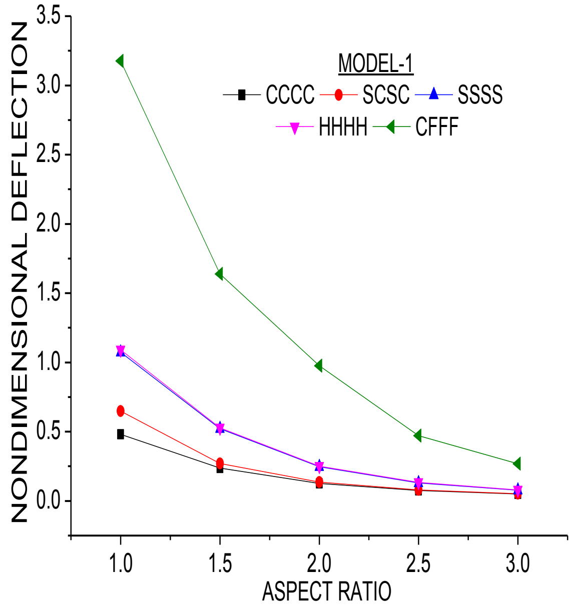


(*e*)

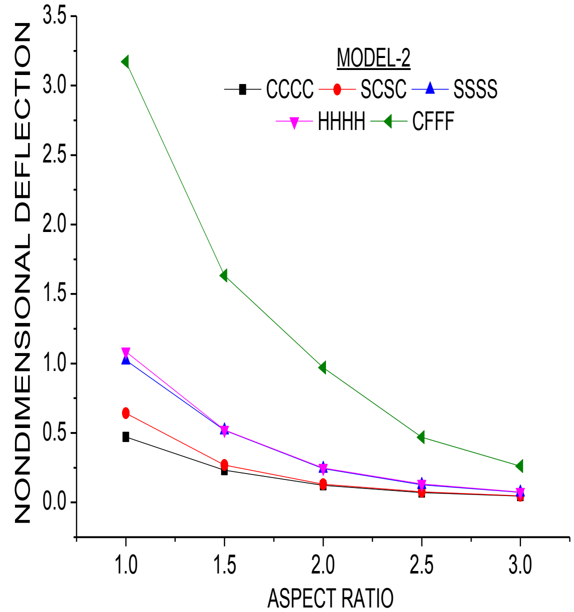
Figure 3.12 (*a*)-(*e*): Deformed shape of five layered cross-ply composite laminated plate under UDL for varying aspect ratio

3.7.2 Effect of support condition on static response of laminated composited plate

It is well known that the support conditions are essential to avoid any rigid body motion as well as to reduce the unknown variable from the final equilibrium equation. In this example, the effect of different support conditions on the nondimensional deflection ($\bar{w}$) of five layered cross-ply composite plate under UDL with a/h=10 and M2 material properties as in Table 3.2 has been investigated for five different aspect ratios (a/b=1, 1.5, 2, 2.5 and 3). It is a well-known fact that with the increase in the number of constraint degrees of freedom, the structure becomes stiffer and the deflection value decreases prominently. For the computational purpose, five support conditions namely CCCC, SCSC, SSSS, HHHH and CFFF are employed to calculate the responses using both the models, Model-1 and Model-2 and are plotted in Figure 3.13. It is interesting to note that the responses are following an increasing trend from CCCC, SCSC, SSSS and HHHH to CFFF.



(*a*)

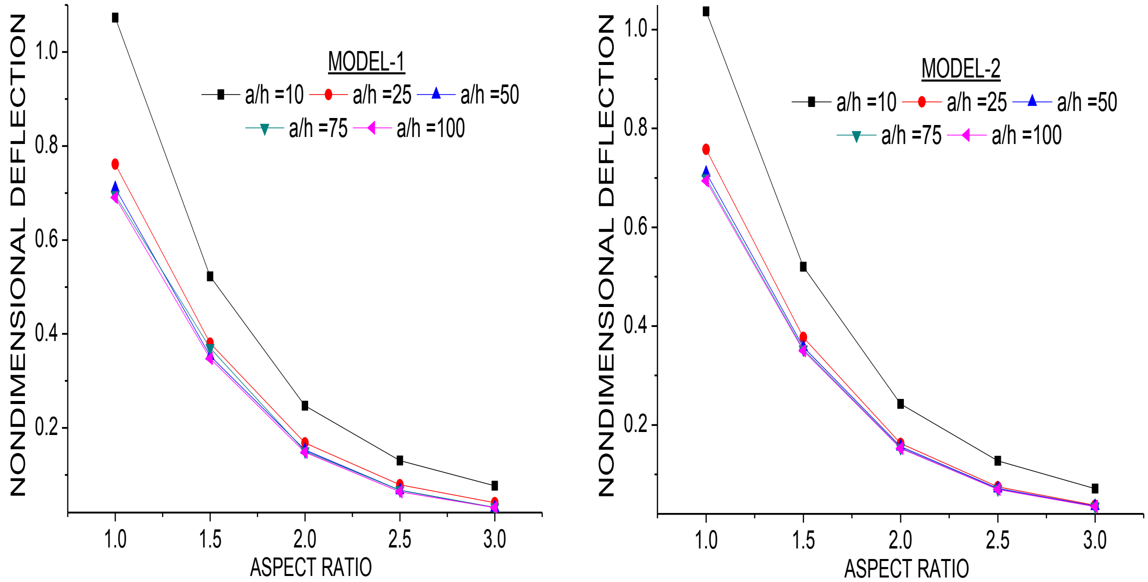


(*b*)

Figure 3.13 (*a*)- (*b*): Nondimensional deflection of five layered cross-ply composite laminated plate under UDL for varying support condition

3.7.3 Effect of thickness ratio on static response of laminated composited plate

The thickness ratio is predominant in stiffness calculation of any structure and with the increase in thickness ratio, the structure becomes thinner. Also, it is true that the thickness ratio of the laminated plate is inversely proportional to the stiffness of the structure. In this example, the nondimensional central deflection ($\bar{w}$) has been examined by employing Model-1 and Model-2 for a five layered simply supported cross-ply symmetric laminated composite plate subjected to UDL with varying thickness ratios (*a*/*h*=10, 25, 50, 75 and 100) and M1 material properties. The flexural responses of the plate are computed for different aspect ratio (*a*/*b*=1, 1.5, 2, 2.5 and 3) and presented in Figure 3.14. It can be clearly observe that the nondimensional deflection decreases as the thickness ratio increases and the responses are within the expected line.



(*a*) (*b*)

Figure 3.14 (*a*)- (*b*): Nondimensional deflection of five layered simply supported cross-ply composite laminated plate under UDL for varying thickness ratio

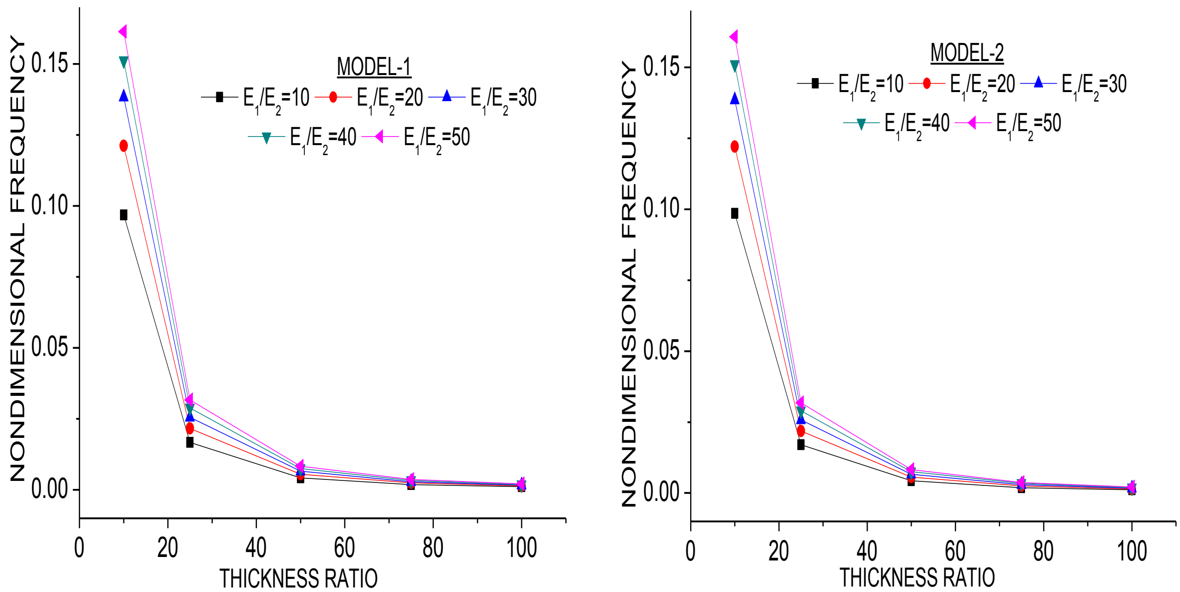
3.7.4 Effect of support condition on in-plane normal stresses

In design point of view, the stress is one of the important parameters in the laminated structure and it has also great physical significance in accordance to the support condition. Therefore, in this example the in-plane normal stresses (σ_{xx} and σ_{yy}) are computed using the Model-1 for square four layered simply supported cross-ply laminated composite plate under UDL with a/h= 20 and M2 material properties as in Table 3.1. The responses are computed for five different support conditions (CFCF, CCCC, SSSS, SCSC and HHHH) and presented in Figure 3.15 with respect to nondimensionalised thickness coordinate (z/h). It can be clearly observed that both the normal stresses are higher for two support conditions (SSSS and HHHH) and lower for the CCCC support. It is well known that as the deflection and stresses of any structure increases or decreases in accordance to their number of constraint and the results are following the same trend.

3.7.5 Effect of modular ratio on free vibration response of laminated composited plate

The impact of varying modular ratio on the free vibration response of a square seven layered simply supported cross-ply laminated composite plate for various thickness ratios (a/h=10, 25, 50, 75 and 100) is analysed in this example by using Model-1 and Model-2. The nondimensional frequency ($\hat{\omega}$) are computed for five different modular ratios (E_l/E_t =

10, 15, 20, 25 and 30) considering M4 material properties and are presented in Figure 3.16. The responses indicate that the nondimensional frequency of the plate increases as the modular ratio increases. It is due to the fact that with the increase in modular ratio, the longitudinal Young's modulus increases which in turn increases the stiffness of the plate directly.

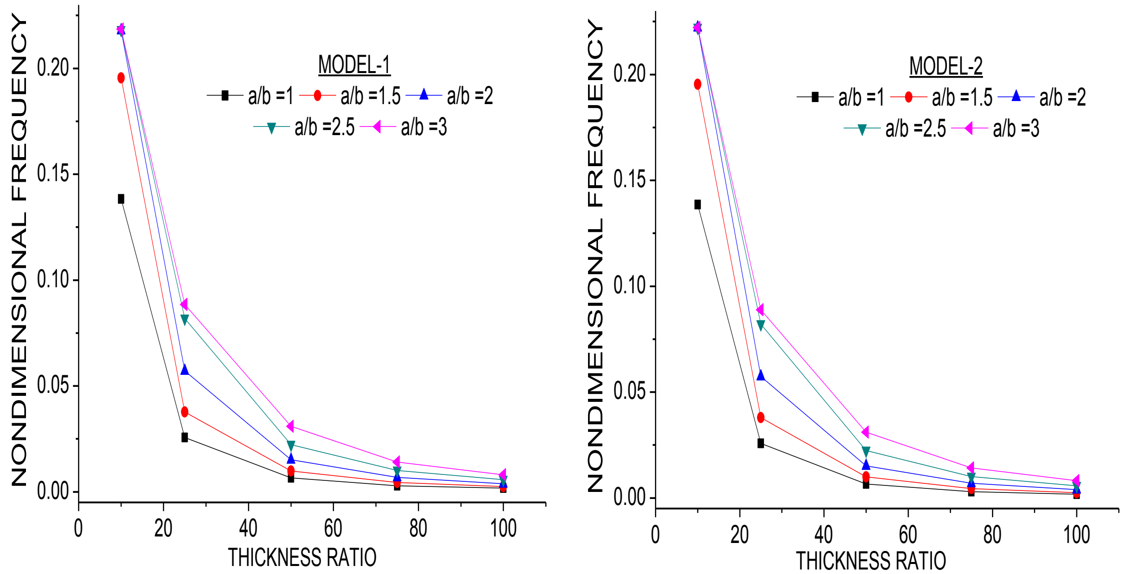


(*a*) (*b*)

Figure 3.16 (*a*)- (*b*): Nondimensional frequency of seven layered square simply supported composite cross-ply laminated plate for varying modular ratio

3.7.6 Effect of aspect ratio on free vibration response of laminated composited plate

In this example, the free vibration response of a seven layered simply supported cross-ply laminated composite plate with E_l/E_t=30 and various thickness ratios (a/h=10, 25, 50, 75 and 100) is examined. The nondimensional frequency ($\hat{\omega}$) is plotted using Model-1 and Model-2 in Figure 3.17 by considering M4 material properties for different aspect ratios (a/b=1, 1.5, 2, 2.5 and 3). The responses specify that the nondimensional frequency of the plate increases as the aspect ratio increases.

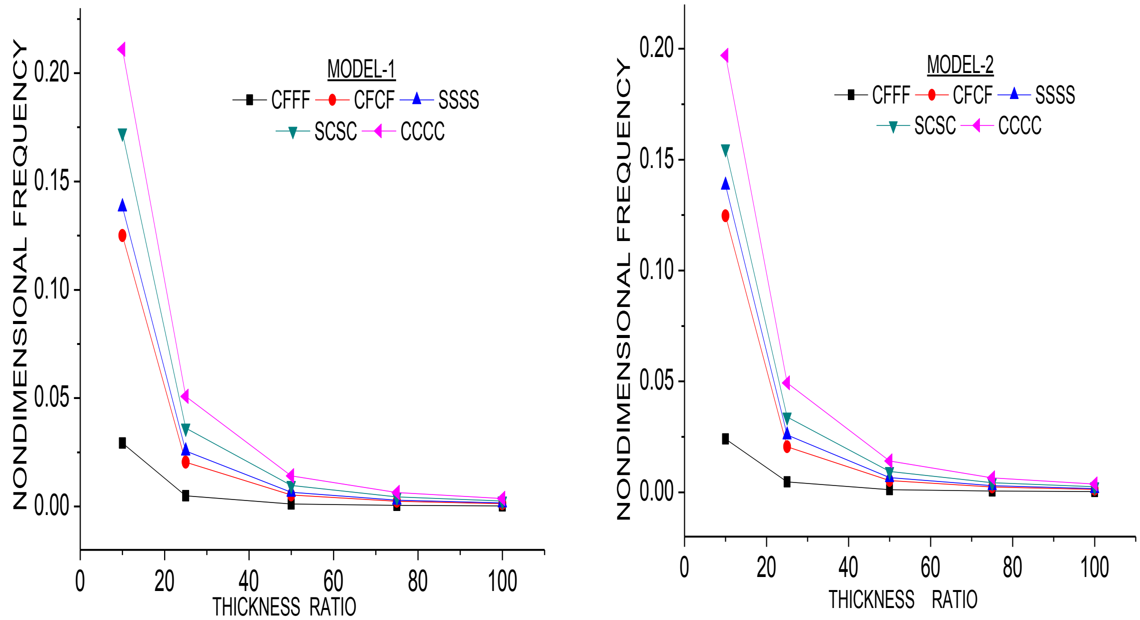


(*a*) (*b*)

Figure 3.17 (*a*)- (*b*): Nondimensional frequency of seven layered simply supported composite cross-ply laminated plate for varying aspect ratio

3.7.7 Effect of support condition on free vibration response of laminated composited plate

Figure 3.18 exhibits the nondimensional frequency ($\hat{\omega}$) of a square seven layered simply supported cross-ply laminated composite plate for various thickness ratios (*a*/*h*=10, 25, 50, 75 and 100). The free vibration response are computed using Model-1 and Model-2 considering E_l/E_t=30, M4 material properties as in Table 3.1 and different support conditions like CCCC, SCSC, SSSS, CFCF and CFFF. The free vibration responses specify that the nondimensional frequency of the plate increases as the number of constrained degrees of freedom increases, i.e., nondimensional frequency response increases in order of CFFF, CFCF, SSSS, SCSC and CCCC.



(*a*) (*b*)

Figure 3.18 (*a*)- (*b*): Nondimensional frequency of seven layered simply supported composite cross-ply laminated plate for varying aspect ratio

3.7.8 Effect of modular ratio on transient response of laminated composited plate

The transient responses of a two layered cross-ply square simply supported composite laminated plate under uniform step load, q_0 with M6 material properties of Table 3.1 are calculated and presented in Figure 3.19. The responses are evaluated for both the higher-order model (Model-1 and Model-2) at three different modular ratios (E_l/E_t = 25, 50 and 75). The responses clearly indicate that as the modular ratio increases, the nondimensional displacement ($\bar{w}$) with respect to time decreases.

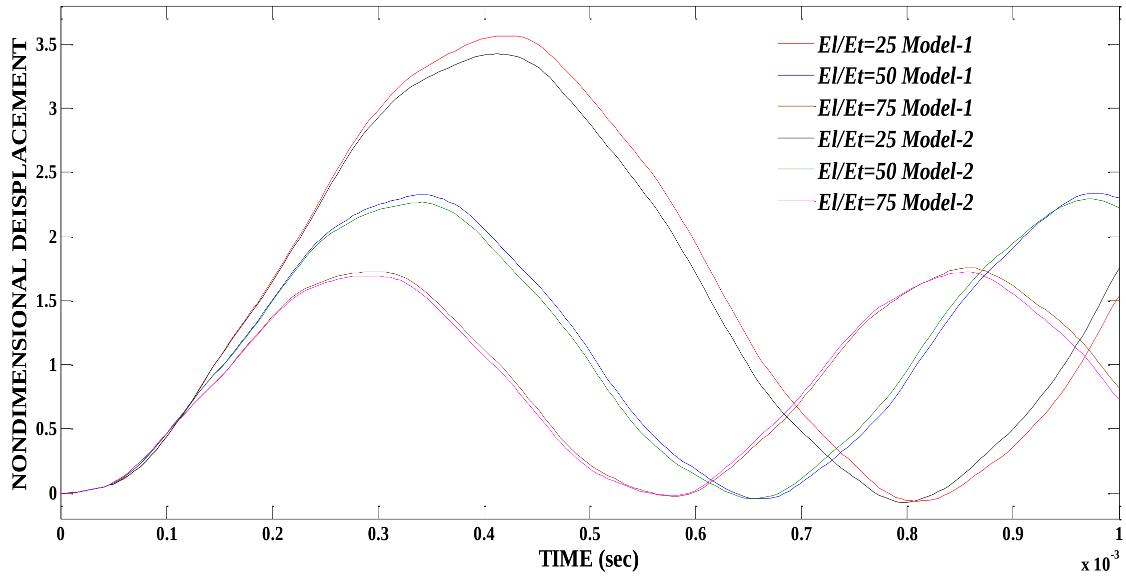


Figure 3.19: Nondimensional deflection of two layered cross-ply simply supported composite laminated plate for varying modular ratio

3.7.9 Effect of aspect ratio on transient response of laminated composited plate

In this example, Model-1 and Model-2 are employed to compute the transient responses of a two layered cross-ply simply supported composite laminated plate with M6 material properties as tabulated in Table 3.1. The nondimensional displacement ($\overline{w}$) are plotted for three different aspect ratios (a/b=1, 1.5 and 2) under uniform step load, q_0 and presented in Figure 3.20. The responses indicate that as the aspect ratio increases, the nondimensional displacement with respect to time decreases.

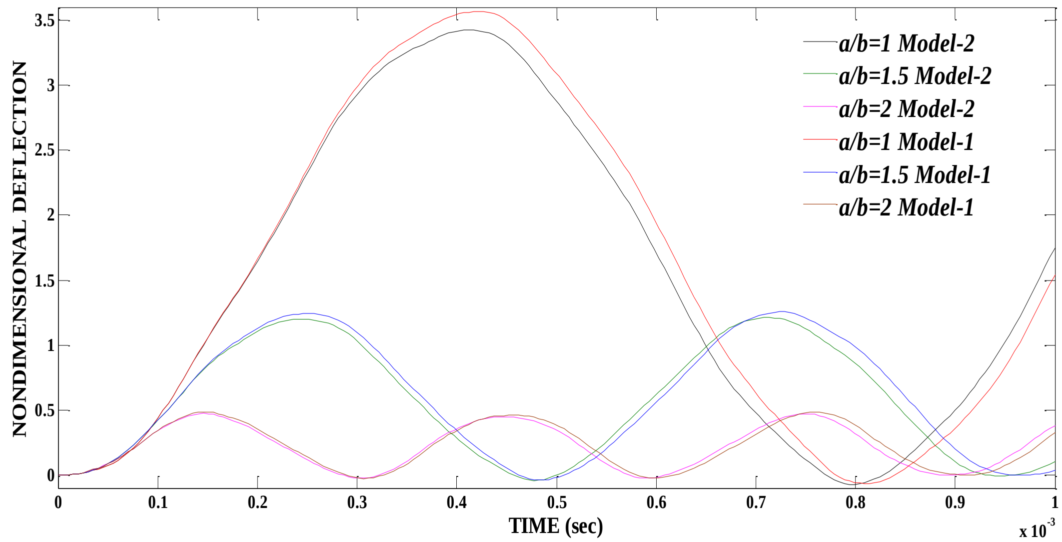


Figure 3.20: Nondimensional deflection of two layered cross-ply simply supported composite laminated plate for varying aspect ratio

3.8 Conclusions

In this chapter, the static and dynamic responses of the laminated composite plate are examined. For the analysis purpose, FE models are developed based on the HSDT kinematics. The governing differential equation for static, free vibration and transient analysis are derived using the variational principle, Hamilton's principle and Newmark's integration, respectively and discretised with the help of suitable isoparametric FE steps. The computer code is developed using the higher-order mathematical models in MATLAB environment for the computation of the responses. Along with that, a simulation model is also developed in ANSYS using APDL code. The convergence behaviour of the present numerical and simulation model has been checked with mesh refinement and also the models have been validated by comparing the responses to that of the available published literature and corresponding experiment. In order to show the efficacy of the present models, the study has been extended for different geometrical parameters of the laminated composite plate. For the same, a number of numerical examples have been solved considering all sort of variations, and the outcomes on the various parametric studies are conversed in the following lines.

- The comparison study of the developed models clearly indicate the necessity of higher order models for better assessment of structural analysis of laminated composite plates.

- The parametric study indicates that the static and dynamic responses of laminated composite plate are greatly depended on the geometrical and material parameters like aspect ratio, thickness ratio and modular ratio and support conditions.
- The nondimensional central deflection parameter increases with decrease in aspect ratio, unconstrained degrees of freedom and thickness ratio.
- The nondimensional natural frequency parameter increases with increase in modular ratio, aspect ratio and unconstrained degree of freedom.
- The nondimensional deflection with respect to time response decreases with increase in modular ratio and aspect ratio.
- Finally, it is noted that both the higher-order kinematic models, i.e., Model-1 and Model-2 are showing the good capability of solving the static, free vibration and transient responses of laminated composite plate. But it is interesting to note that the vibration responses computed using the Model-1 is close to the experimental results. However, the Model-2 is capable to solve the static responses of the laminated structure due to the absence of unrealistic assumption of stretching term through the thickness.

Chapter 4

Static and Dynamic Analysis of Delaminated Composite Plate

4.1 Introduction

In this chapter, the static and dynamic analyses of composite plates with delamination have been investigated using the proposed higher-order models. The repeated cyclic stresses, manufacturing processes, unlike environmental conditions, etc., may cause the layers of a laminated composite plate to detach which is known as delamination. It is an insidious kind of failure, without being distinct on the surface. Delamination leads to significant loss of stiffness which in turn reduces the natural frequency of the laminate and it may lead to resonance if the resulted value is close to the working frequency value. Therefore, the monitoring of delamination in composite material is quiet inevitable to model these complex structural problems precisely. In Section 4.2, the governing equation of the static, free vibration and dynamic analysis of composite plate with delamination are provided along with its solution steps. Section 4.3 presents comparison study of the developed higher order models for the analysis of static, free vibration and transient analysis of the delaminated composite plates with responses obtained from previously published literature and succeeding experimental studies. In continuation, the robustness and the efficacy of the presently developed models have been revealed through the comprehensive parametric study. The effects of various geometrical and material parameters, and the support conditions on the structural responses of the delaminated composite plates are inspected. Finally, this chapter is summarised with the concluding remarks in Section 4.4

4.2 Governing Equation and Solution Methodology

The higher-order models for the structural analysis of composite plates have been modelled for delamination in Section 2.7 in the very next section, continuity conditions were specified. The final form of equilibrium equation for static, free vibration and dynamic analysis of laminated composite plate is derived using the variational principle, Hamilton's principle and Newmark's integration, respectively as discussed in Section 2.9. Also, the detailed solution steps used in the analysis procedure are stated in Section 2.11. For the analysis purpose, customised homemade computer codes for the above mentioned models have been developed in MATLAB environment to compute the desired responses.

4.3 Results and Discussions

The finite element solutions of the static and dynamic behaviour of laminated composite plate have been computed with the help of homemade computer code developed in MATLAB environment using the proposed higher-order models as discussed in Chapter 2. The results are computed using both the models (Model-1 and Model-2) and compared with previously available literature and succeeding experiments. Subsequently, wide variety of numerical examples are solved to examine the accuracy of the proposed models considering different material and geometrical parameters of the delaminated plate. The effect of different combinations of parameters such as the thickness ratio (a/h), the aspect ratio (a/b), the modular ratio (E_l/E_t) and the support condition on the static and dynamic behaviour of composite plate with delamination along with the size and location of the delamination are also discussed in detailed.

The static and dynamic responses of delaminated composite plates were calculated considering different for the structural analysis of composite plates have been modelled for delamination in Section 2.7 in the very next section, continuity conditions were specified. The final form of equilibrium equation for static, free vibration and dynamic analysis of laminated composite plate is derived using the variational principle, Hamilton's principle and Newmark's integration, respectively as discussed in Section 2.9. Also, the detailed solution steps used in the analysis procedure are stated in Section 2.11. For the analysis purpose, customised homemade computer codes for the above mentioned models have been developed in MATLAB environment to compute responses are computed for validation purpose by utilizing various geometrical and material configurations as presented in Table 4.1.

Table 4.1: Material Properties of delaminated composite plates

Properties	MATERIAL-1 (M1)	MATERIAL-2 (M2)	MATERIAL-3 (M3)	MATERIAL-4 (M4)
a	0.25m	0.5m	0.25m	0.15m
b	0.25m	0.5m	0.25m	0.15m
h	0.00212m	b/100	0.05m	0.002m
E_l	1.32×10^2GPa	172.5GPa	$25\times E_2$	7.366GPa
E_t	5.35GPa	6.9GPa	1×10^9GPa	5.568GPa
E_z	E_t	E_t	E_t	5.568GPa
G_{lt}	2.79GPa	3.45GPa	$0.5\times E_t$	2.79GPa
G_{tz}	2.79GPa	1.38GPa	$0.5\times E_t$	1.395GPa
G_{lz}	1.395GPa	3.45GPa	$0.2\times E_t$	2.79GPa
υ_{lt}	0.291	0.25	0.25	0.17
υ_{lz}	0.291	0.25	0.25	0.17
υ_{tz}	0.3	0.25	0.25	0.17
ρ	1446.2 kg/m^3	1600 kg/m^3	1 kg/m^3	1100 Kgm^{-3}

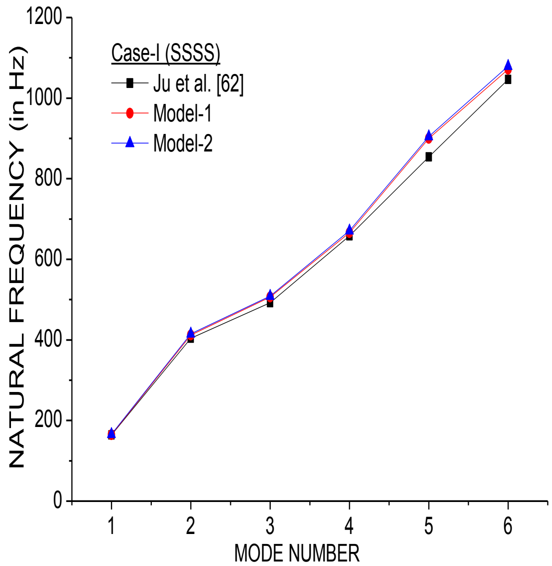
4.4 Validation Study of Dynamic Response

The authenticity and constancy of the present higher order models have been checked by comparing the dynamic responses of the composite plates with delamination with those available published literature. The free vibration response as well as the transient response of the delaminated composite plate were found out by utilizing the numerical models, Model-1 and Model-2 and then were compared with the responses available in published literature.

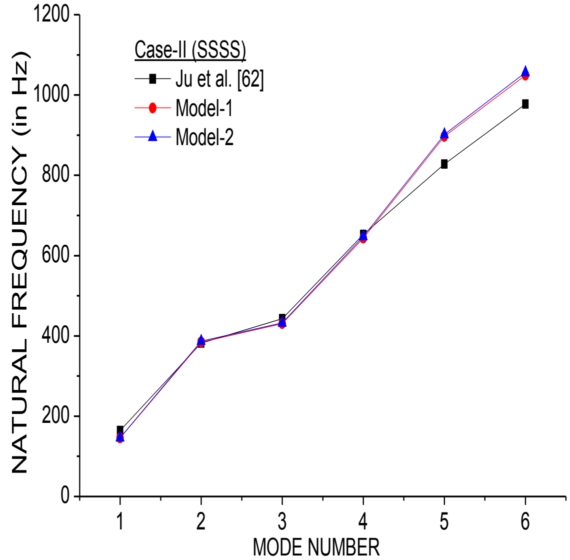
4.4.1 Comparison study of the free vibration analysis of delaminated composite plate

The free vibration responses of an eight layered square composite plate of $[0^0/-45^0/45^0/90^0]_s$ are computed using M1 material and geometrical properties as in Table 4.1. The first six natural frequencies of the composite plate are computed for three cases of delamination, Case-I, Case-III and Case-IV. The responses are computed for different support conditions (FFFF, CFFF, SSSS and CCCC) and compared with Ju et al. [62] and presented in Figure

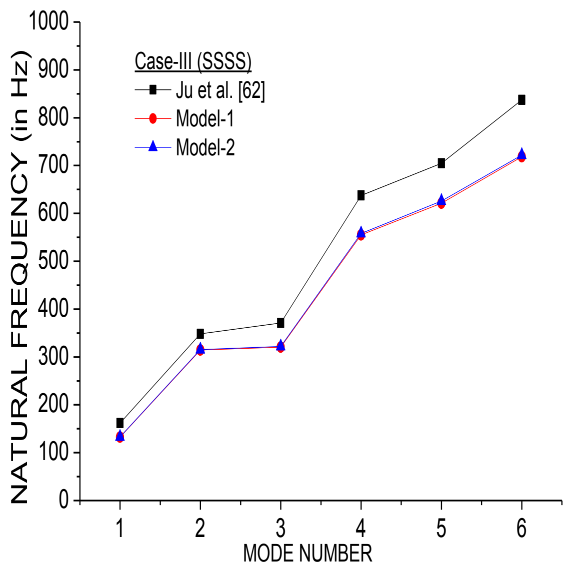
4.1 (*a*)-(*l*). It can be seen that the responses of proposed higher order models, Model-1 and Model-2 are showing decent agreement with that of the references for every case of delamination as presented in the figure.



(*a*)



(*b*)



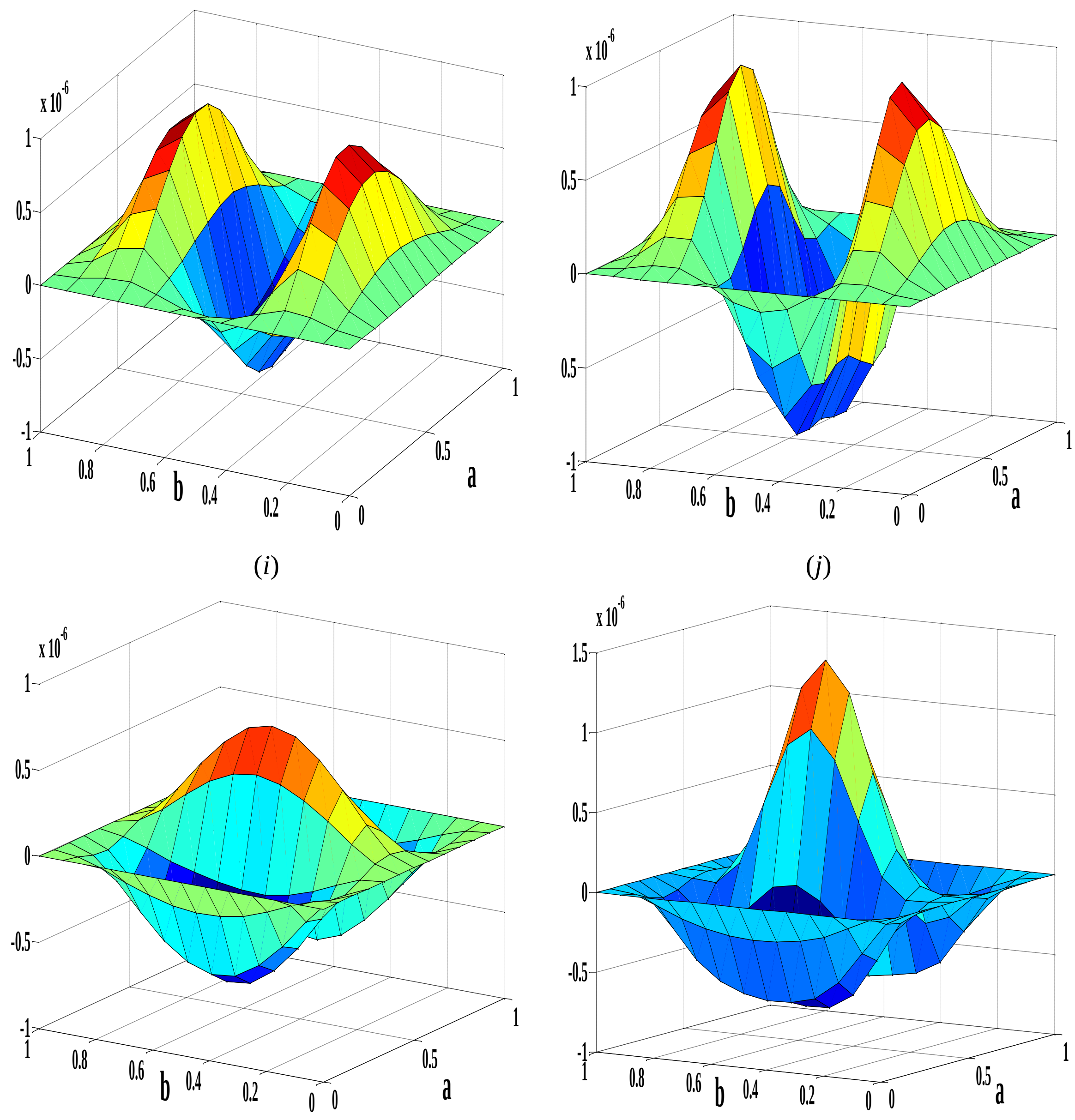
(*c*)

Figure 4.1 (*a*)-(*l*): Natural frequency of eight layered square composite plate with three cases of delamination

Also, the first six modes of the natural frequency of the composite plate with clamped boundary conditions are computed for two cases i.e.- Case-I and Case-III utilizing one of the higher order model, Model-1 and plotted in Figure 4.2. It can be clearly understood from the figure that delamination affects the higher mode shapes much more than the lower ones.

Mode No. | Case-I | Case-III

5 | 6



(*i*) (*j*) (*k*) (*l*)

Figure 4.2 (*a*)-(*l*): Mode shapes of fisrt six natural frequency of clamped composite plate for Case-I and Case-III type of delamination

4.4.2 Comparison study of the free vibration analysis of delaminated composite plate

The natural frequency of a square twenty layered $(\pm\theta)_{10}$ cantilever Carbon/Epoxy composite plate have been calculated by employing the higher order models, Model-1 and Model-2. By employing the developed higher order models, Model-1 and Model-2, the transient responses of a delaminated composite plate with simply supported and hinged support conditions have also been computed. The responses are computed for the two layered square cross-ply composite plate under uniform step load, with material and geometrical properties corresponding to M3 of Table 4.1. These responses are computed for two cases of delamination (Case-I and Case-IV) and presented in Figure 4.3 which are compared with Parhi et al. [75]. It can be clearly observed that the present results are showing good agreement with that of the reference.

4.4.3 Comparison study of the transient analysis of delaminated composite plate

By employing the developed higher order models, Model-1 and Model-2, the transient responses of a delaminated composite plate with simply supported and hinged support conditions have also been computed. The responses are computed for the two layered square cross-ply composite plate under uniform step load, $q_0 = 10N/cm^2$ with material and geometrical properties corresponding to M3 of Table 4.1. These responses are computed for two cases of delamination (Case-I and Case-IV) and presented in Figure 4.3 which are compared with Parhi et al. [75]. It can be clearly observed that the present results are showing good agreement with that of the reference.

Figure 4.3 (*a*): Central displacement of two layered square simply supported cross-ply composite plate with Case-I and Case-IV of delamination

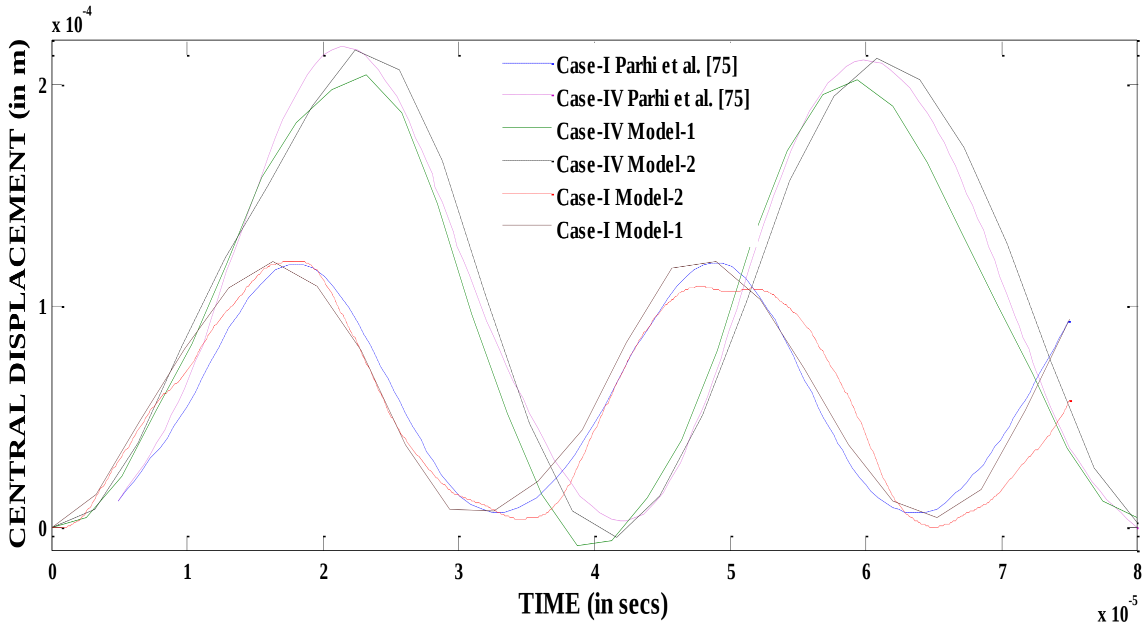


Figure 4.3 (*b*): Central displacement of two layered square hinged cross-ply composite plate with Case-I and Case-IV of delamination

4.5 Experimental Studies

Furthermore, free vibration responses of woven Glass/Epoxy plates are found out experimentally and subsequently compared with those of the developed numerical models, Model-1 and Model-2 with the purpose of building more confidence on the these models. The woven Glass/Epoxy composite plate is fabricated using hand layup method with delamination at any desired location. Material property and subsequently the free vibration responses of the fabricated plate were found out and compared with the responses of the proposed higher order models.

4.5.1 Fabrication of composite plate with delamination

Glass/Epoxy composite plate specimens were fabricated using free vibration responses of woven Glass/Epoxy plates are found out experimentally and subsequently compared with those of the developed numerical models, Model-1 and Model-2 with the purpose of building more confidence on the these models. The woven Glass/Epoxy composite plate is fabricated using hand layup method with delamination at any desired location. Material property and subsequently the free vibration responses of the fabricated plate were found out and compared with the responses of the proposed higher order models as discussed in Section 3.6.1 for the laminated composite plates as shown in Figure 4.4(a). Further, the responses are also computed using a simulation model developed in ANSYS environment with the help of APDL code for the laminated composite structure. The present ANSYS model is developed and discretised using an eight node serendipity element (SHELL281) with six degrees of freedom at each node. In this simulation model, the mid-plane kinematics of the plate is governed by the FSDT. The brief description of solution procedure in ANSYS platform are as follows:

4.5.2 Material property evaluation

The mechanical properties say, Young's modulus, shear modulus and Poisson's ratio in principal material directions of the delaminated composite plate The free vibration behaviour of the woven Glass/Epoxy plate with delamination was found out experimentally with the help of modal test analysis using PXIe-1071 instrument at NIT Rourkela. The natural frequencies of the delaminated plate are acquired with LABVIEW software and details are test conducted via UTM INSTRON 1195 as discussed in Section 3.6.1 for the laminated composite plates. The material properties so obtained are presented as M4 material properties in Table 4.1.

4.5.3 Free vibration response

The free vibration behaviour of the woven Glass/Epoxy plate with delamination was found out experimentally with the help of modal test analysis using PXIe-1071 instrument at NIT Rourkela. The natural frequencies of the delaminated plate are acquired with LABVIEW software and details are in Section 3.6.3. The experimental responses are compared with those of the present numerical results computed using developed mathematical models, Model-1 and Model-2 and presented in Table 4.3. The comparison study clearly indicates that the present HSDT models are inevitable for the accurate analysis of the laminated structures.

4.6 Numerical Illustrations

It is obviously evident from the outcomes as discussed in the earlier sections that the present higher-order mathematical models are essential and inevitable for the structural analysis of composite plates with and without delamination. Now, considering different material and geometrical properties of the delaminated composite plates, the efficiency of the presently developed finite element models is established through by solving a variety of numerical problems. Static, free vibration and transient responses are computed for different geometrical parameter (the thickness ratios, the modular ratios and the aspect ratios) and the support conditions are examined and discussed in detailed in the following subsections. The responses are computed for eight layered square symmetric composite plate with [0^0/-45^0/45^0/90^0] s configuration unless otherwise stated.

4.6.1 Effect of delamination size on the static response of delaminated composite plate

It is a fact that the size of delamination plays a crucial role on the structural strength of the laminated structure and the central deflection parameter is also affected largely due to that. In this example, the central deflection of the plate with simply supported boundary condition is examined using M1 material properties as tabulated in Table 4.1 under different UDL (*p*=10, 20, 30, 40, 50, 60, 70, 80, 90 and 100) with the help of the higher order models (model-1 and Model-2) and presented in Figure 4.5. In the present investigation, all the cases of delamination are considered namely Case-I, Case-II, Case-III, Case-IV and Case-V. The responses are computed using both the proposed higher order models (Model-1 and Model-2) and it was observed that the central deflections of the plate increases as the delamination size increase.

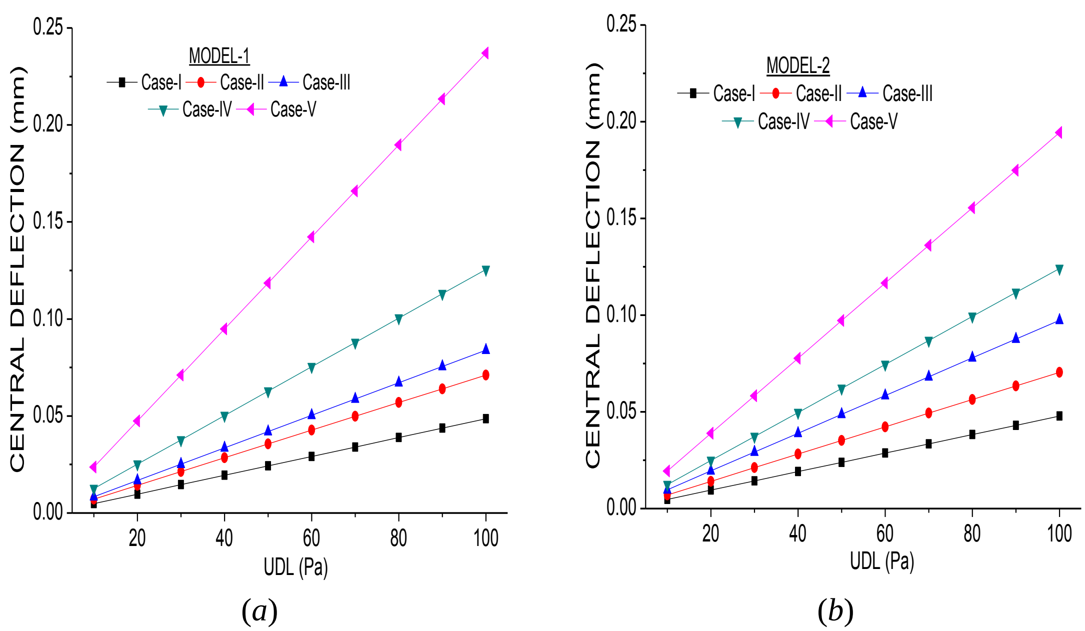
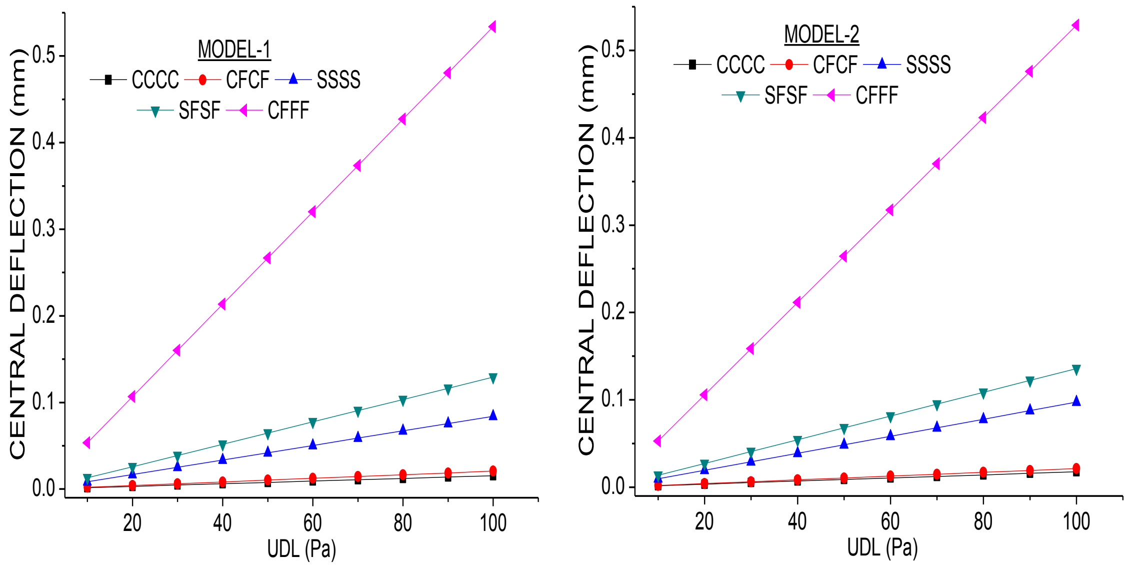


Figure 4.5 (*a*)-(*b*): Central deflection of eight layered square simply supported composite plate for various sizes of delamination

4.6.2 Effect of support condition on the static response of delaminated composite plate

The central deflection of the composite plate with Case-III type of delamination under different UDL (*p*=10, 20, 30, 40, 50, 60, 70, 80, 90 and 100) is computed using M1 material properties as in Table 4.1 in this example. In general, the support conditions constrained some degrees of freedom of the structure and it is well-known fact that with the increase in the number of constraint the structure becomes stiffer and the deflection value decreases prominently. The effect of five support conditions (CCCC, CFCF, SSSS, SFSF and CFFF) on the delaminated plate is computed using both the models (Model-1 and Model-2) and presented in Figure 4.6. It is interesting to note that the responses are following an increasing trend progressively in the order of CCCC, CFCF, SSSS, SFSF and CFFF.

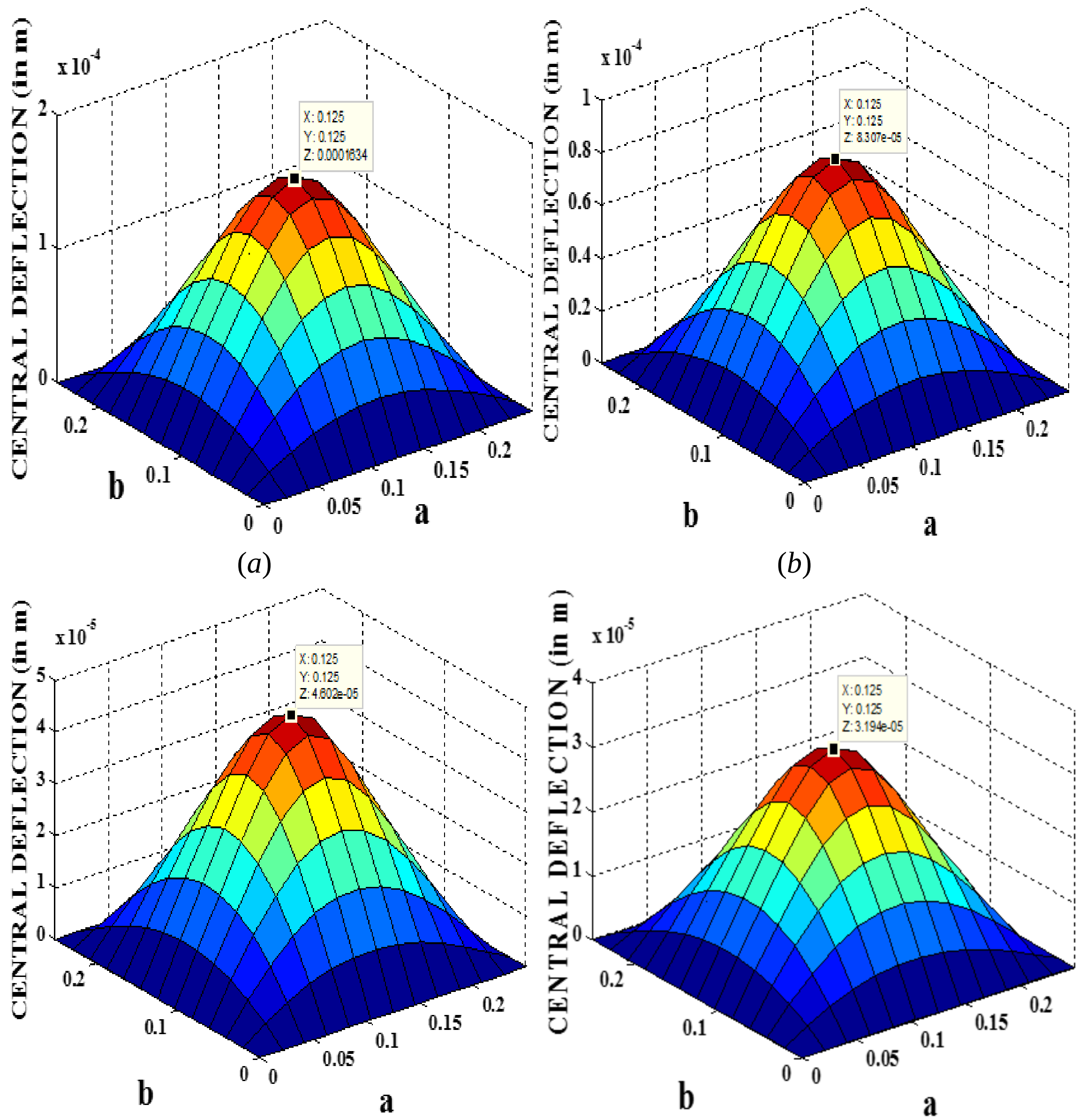


(*a*) (*b*)

Figure 4.6 (*a*)-(*b*): Central deflection of eight layered square composite plate with Case-III type of delamination for varying support condition

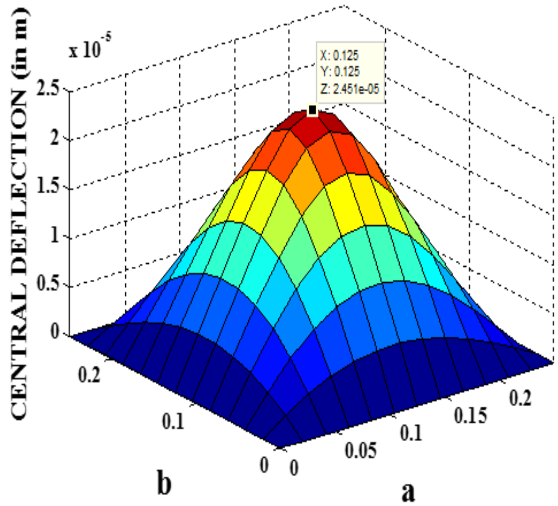
4.6.3 Effect of modular ratio on the static response of delaminated composite plate

In this example, the Model-1 and the Model-2 are employed to examine the influence of different modular ratios on the static response of the delaminated composite plate with simply supported boundary condition considering M1 material properties as in Table 4.1 and p=100. The deformed shapes of the plate with Case-III type of delamination are computed for five different modular ratios (E_l/E_t = 10, 15, 20, 25 and 30) and presented in Figure 4.7. It can be clearly understood from the deformed shape that the central deflections of the plate decreases as the modular ratio increase and the results are within the expected line. It is due to the fact that the longitudinal Young's modulus increases as the modular ratios increase and this in turn increases the stiffness of the plate structure directly.


CENTRAL DEFLECTION (in m)
x 10^-4
X: 0.125
Y: 0.125
Z: 0.0001634
b
a
X: 0.125
Y: 0.125
Z: 8.307e-05
x 10^-5
X: 0.125
Y: 0.125
Z: 4.602e-05
X: 0.125
Y: 0.125
Z: 3.194e-05

(*a*) (*b*)

(*c*) (*d*)



(*e*)

Figure 4.7 (*a*)-(*e*): Central deflection of eight layered square composite plate with Case-III type of delamination for varying modular ratio

4.6.4 Effect of delamination location on the free vibration response of delaminated composite plate

Also, the natural frequency of the composite delaminated plate with simply supported boundary condition is computed varying the location of delamination. The delamination is assumed to be located in various locations of the laminated plate as shown in Figure 4.8. It can be seen from the Figure 4.9 that the natural frequencies decrease as the delamination is shifted from the mid-plane i.e., the delamination at the mid-plane of the laminated structure affect the natural frequency significantly than the other three interfaces. It is also observed that the frequency responses are very close to the laminated case for the location 4 i.e., it doesn't affect the frequency responses much. This can be contributed to the fact that delamination at the inner interfaces may cause a greater decrease in the global stiffness than at outer interfaces.

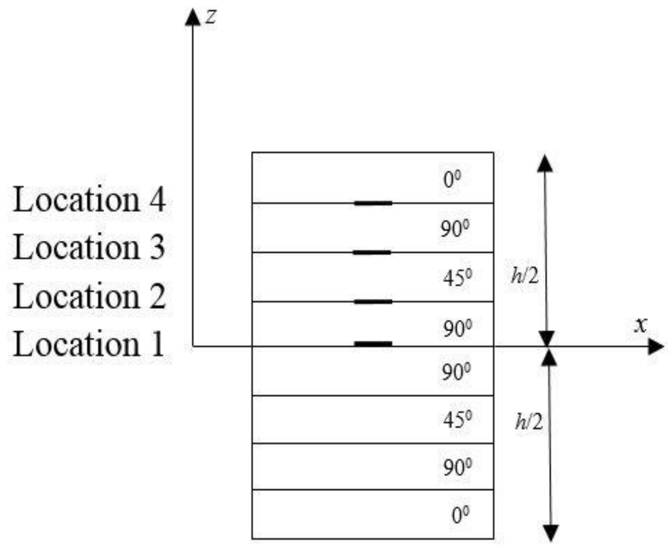
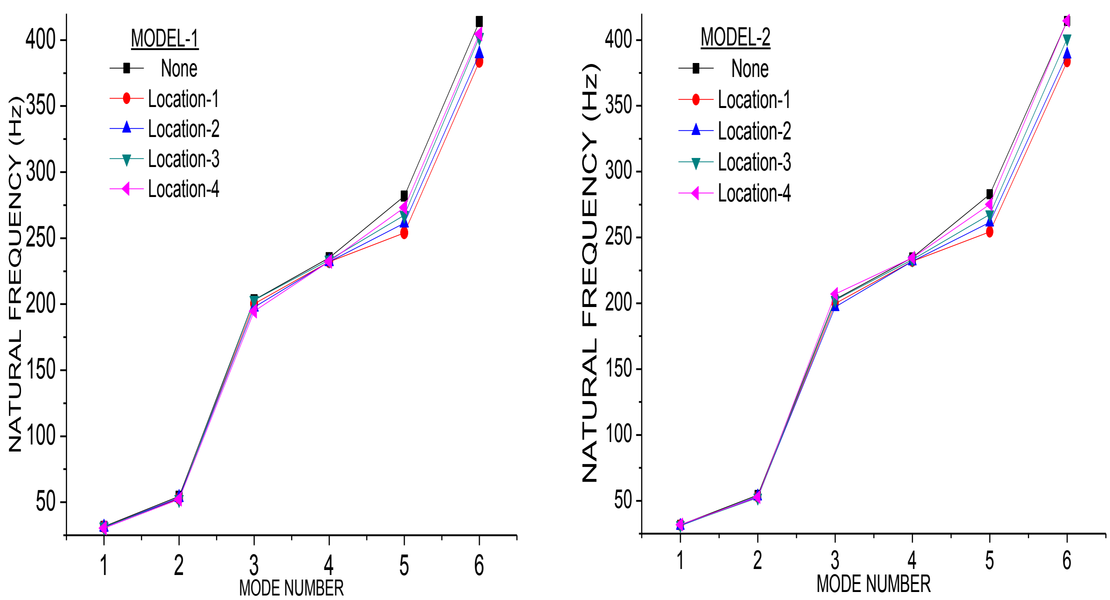


Figure.4.8: Ply configuration of composite plate showing various location of delamination



(*a*) (*b*)

Figure 4.9 (*a*)-(*b*): Natural frequency of eight layered square simply supported composite delaminated plate for various location of delamination

4.6.5 Effect of thickness ratio on the free vibration response of delaminated composite plate

It is well known that the thickness ratio of any structure or the structural component increases then the structure becomes thinner and the structural stiffness of the laminated panel is inversely proportional to that. The free vibration response has been examined using the proposed higher-order models (Model-1and Model-2) for the plate with Case-III type of delamination for various thickness ratios (*a*/*h*=20, 40, 60, 80 and 100). The first six natural frequency of the plate with simply supported boundary condition and M1 material properties are computed and presented in Figure 4.10. It is clearly observed that the natural frequency decreases as the thickness ratio increases and the responses are within the expected line.

4.6.6 Effect of aspect ratio on the free vibration response of delaminated composite plate

The aspect ratio (*a*/*b*) of any structural component is a crucial factor for determination of the stiffness and the stability, mainly for the case of laminated structures. Model-1 and Model-2 are engaged to find out the natural frequency of the plate with simply supported boundary condition having M1 material properties as in Table 4.1 and Case-III type of delamination located at the middle of the plate at mid-plane of the structure. The first six natural frequency are plotted for five different aspect ratios (*a*/*b*=1, 1.5, 2, 2.5 and 3) are presented in Figure 4.11. It can be established that with the increase in aspect ratio, the natural frequency of a delaminated plate increases.

4.6.7 Effect of thickness ratio on the transient response of delaminated composited plate

In this example, the transient responses of a two layered hinged square cross-ply delaminated composite plate are plotted in Figure 4.12 considering M1 material properties as tabulated in Table 4.1. Model-1 and also Model-2 are employed to plot the responses for the delaminated plate under uniform step load, $q_0 = 10N/cm^2$ with three different thickness ratios (*a*/*h*=6, 8 and 10) with Case-IV type of delamination. From the figure, it can be concluded that the central deflection increases as the thickness ratio increases.

4.6.8 Effect of modular ratio on the transient response of delaminated composited plate

The consequence of varying modular ratio on the transient responses of a two layered square cross-ply delaminated composite plate with hinged type of support conditions with M5 material properties are plotted in this example. The plate is considered to be under uniform step load, $q_0 = 10N/cm^2$ with Case-IV type of delamination. The responses are plotted in Figure 4.13 for different modular ratios (E_l/E_t=25, 50 and 100) using both Model-

1 and Model-2. It can be clearly observed that as the modular ratio increases, the central deflection of the plate decreases.

4.7 Conclusions

In this chapter, the static and the dynamic responses of composite plates with delamination are investigated. The desired static, free vibration and transient responses are computed numerically with the help of higher-order FE code developed in MATLAB environment. The governing differential equation for static, free vibration and transient analysis are derived using the variational principle, Hamilton's principle and Newmark's integration, respectively. The responses of the higher order models are validated by comparing them with those available in published literature. Moreover, the composite plate with delamination is fabricated with the help of hand lay-up method and the responses are obtained experimentally for the comparison purpose. In order to justify the accuracy for the analysis of the laminated structure with desired delamination using proposed mathematical models, the study has been extended for different geometrical parameters of the delaminated composite plate. For the same, a number of numerical examples have been solved considering different variations, and the outcomes on the various parametric studies are discussed in the following lines.

- The comparison study of the developed models clearly indicate the necessity of higher order models for better assessment of structural analysis of laminated structure with delamination.
- The parametric study indicates that the static and dynamic responses are greatly depended on the size and location of the delamination and also on the geometrical and material properties (aspect ratio, thickness ratio and modular ratio) and support conditions.
- With increase in the size of the delamination, the central deflection of the delaminated composite plates also increases.
- The central deflection of the delaminated plate decreases with increase in constrained degrees of freedom and modular ratio.
- The presence of delamination at the mid-plane of the laminated structure affect the natural frequency of the plate significantly than the other three interfaces.
- The natural frequency parameter of delaminated plate decreases with decrease in aspect ratio and increase in thickness ratio.
- The central deflection with respect to time response of the delaminated plate decreases with increase in modular ratio and decrease in thickness ratio.
- Finally, it is seen that the Model-1 and Model-2 both are showing the good capability of solving the static, free vibration and transient responses of composite plate with

delamination. But it is interesting to note that the vibration responses computed using the Model-1 is close to the experimental results.

Chapter 5

Closure

5.1 Concluding Remarks

In this work, three general mathematical models for laminated composite plates with/without delamination are proposed and developed to investigate the static, free vibration and transient behaviour. The higher-order models, i.e., Model-1 and Model-2 are developed using computer code developed in MATLAB R2012b environment based on the HSDT mid-plane kinematics. The domain discretised with the help of a nine noded isoparametric Lagrangian element with nine and ten degrees of freedom per node, respectively. In addition to that, a simulation model, Model-3 is developed in commercial FE package (ANSYS 15.0) based on ANSYS parametric design language (APDL) code in order to further substantialize the present analysis.

The final forms of equilibrium equations for the static, free vibration and transient analysis are derived using the variational principle, Hamilton's principle and Newmark's integration, respectively and solved subsequently through a suitable finite element approach in conjunction with the direct iterative method. The convergence study of the present numerical and simulation models have been performed through various illustrations for laminated composite plates and then validated by comparing the responses with those available in earlier published literature. The comparison study of the present analysis indicates the necessity and the importance of the presently proposed higher order models for the structural responses. Therefore, the higher-order models are employed to study the static and dynamic analysis of composite plates with delamination. Furthermore, experimental study are also conducted to determine the static and dynamic responses of both laminated and delaminated composite plates and compared with the developed models. The influence of the various geometrical (the thickness ratio, the curvature ratio, the aspect ratio), the support conditions and size and location of delamination on the static, free vibration and transient responses of laminated composite plates with and without delamination. Based on the outcomes of previous chapters (Chapters 3 and 4), the following concluding remarks are drawn:

- It is evident that the mathematical models based on the HSDT mid-plane kinematics are more realistic for the small strain and large (finite) deformation problems.
- The convergence studies of different static and free vibration problems confirm the consistency and the stability of the present FE models. It has been also found that a (6×6) mesh is sufficient for the static problems whereas for the free vibration problems, the models converge well at (6×6) mesh.
- The comparison studies with earlier published literature reveal the effectiveness and the necessity of the presently developed higher order models for different problems such as the bending, free vibration and the transient analysis of the laminated composite plates.
- The accuracy of the presently proposed models is also justified by the experimental studies. The static and free vibration responses of woven Glass/Epoxy, GFRP and CFRP laminated plates computed via three point bend test and modal test, respectively matches well with responses of the developed models
- The numerous numerical illustrations clearly exhibit the robustness and the stability of the present FE models. It is also noted that the static, free vibration and transient behaviour of the laminated composite plates greatly depend on the geometrical and material parameters and the support conditions.
- The static responses of a laminated composite plate increase when the aspect ratio of the plate decreases, the number of unconstrained degree of freedom decreases and thickness ratio also decreases.
- The free vibration response of a laminated composite plates increases when the modular ratio increases, the aspect ratio increases and when the number of unconstrained degree of freedom increases.
- The transient response of the laminated composite plates decreases when the modular ratio and aspect ratio of the plate increases.
- Also for the delaminated composite plate, comparison studies of the static, free vibration and transient responses of higher-order models with earlier published literature expose the efficacy and the inevitability of these developed models.
- The experimental studies for the free vibration responses of the fabricated woven Glass/Epoxy plate with delamination and its comparison with the developed models also exhibit the exactness of the developed models.

- The consistency of the proposed higher-order models is exhibited by solving numerous numerical illustrations. It is clearly exhibited that the static and dynamic response of the delaminated composite plates is greatly affected by size and location of delamination and also by the geometrical and material parameters and the support conditions of the plate.
- With the increase in the size of the delamination, the static responses of the delaminated composite plates increase. Also, the central deflection of the delaminated plate decreases with increase in constrained degrees of freedom and modular ratio.
- The presence of delamination at the mid-plane of the laminated structure affects the free vibration responses of the delaminated plate significantly than the other three interfaces. The free vibration response of delaminated plate decreases with a decrease in aspect ratio and increase in thickness ratio.
- Finally, the transient response of the delaminated plate decreases with increase in modular ratio and decrease in thickness ratio.

5.2 Significant Contributions of the Thesis

In this work, first time two higher-order models have been proposed for the dynamic analysis of laminated composite panel structure with seeded delamination. The study is also extended to compute the desired responses with the help of simulation model developed using commercial FE package (ANSYS). This is also first time the responses of laminated composite with and without delamination are validated with subsequent experiments. For the computational purpose a homemade generalised computer code is developed in MATLAB (R2012) environment. In addition to that, the simulation model is developed APDL code for both the laminate and delaminated case. The proposed higher-order models are validated by comparing the responses with those available published literature and subsequent experiments.

Finally, it is understood from the previous discussions that the mathematical models developed in the framework of the HSDT would be useful for more accurate analysis of laminated and delaminated composite structures to understand their static, free vibration and transient responses. On the other hand, it is observed that the present developed FE model in ANSYS environment is also capable of solving static, free vibration and transient behaviour easily with less computational time. And hence, the present analysis would be useful for the practical design of the laminated structure.

5.3 Future Scope of the Research

- The present study has been done for the linear cases only and it can be extended for the nonlinear analysis of laminated and delaminated composite structures.
- The present study can be extended to investigate the nonlinear forced/damped vibration and thermo-mechanical post-buckling behaviour of laminated and delaminated composite structures by taking temperature dependent material properties based on the nonlinear mathematical model.
- By extending the present model, a nonlinear mathematical model can be developed to study the behaviour of laminated and delaminated composite structures in thermal and/or hygrothermal environment.
- The smart (piezo, shape memory alloys and magnetostrictive) materials can be incorporated in the nonlinear model to study the effects of material and geometrical parameters.
- It will be interesting to study the flutter characteristics considering the aerodynamic and acoustic loading that frequently arises in the practical cases.
- The optimization study can also be performed for the delamination study to propose the role of size and position on the responses of the laminated structure.

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Appendix

Nodal shape function as appeared in Eq. (2.6)

$$[N_i]=\begin{bmatrix} N_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix}_{i=1\,to\,9} \tag{B.1}$$

Thickness co-ordinate Matrices as appeared in Eq. (2.7)

$$[T]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & z^3 \end{bmatrix} \tag{B.2}$$

Parameters of Newmark's integration as appeared in Eq. (2.30)

$\delta \geq 0.50$, $\alpha \geq 0.25(0.5+\delta)^2$, $a_0 = 1/\alpha\,\Delta t^2$, $a_1 = \delta/\alpha\,\Delta t$, $a_2 = 1/\alpha\,\Delta t$,

$$a_3 = 1/2\alpha - 1,\; a_4 = \delta/\alpha - 1,\; a_5 = \Delta t/2(\delta/\alpha - 2),\; a_6 = \Delta t(1-\delta),\; a_7 = \delta\Delta t \tag{B.3}$$

Vitae

Sushree Sasmita Sahoo, the author, was born on 1st May 1991 in Bhubaneswar, Odisha, India. She has passed the Higher Secondary (10+2) Examination from D.A.V Public School, Unit-VIII, Bhubaneswar under the CBSE with distinction in 2009 and graduated in Mechanical Engineering with a first class degree from Indira Gandhi Institute of Technology(an autonomous college of Govt. of Odisha), Sarang, Odisha, India in the year 2013. On completion of her graduation, she planned to continue his career in the field of research and joined M.Tech (Research) Program in NIT Rourkela, Odisha, India with specialization in Machine Design and Analysis in 2014. The author's current research interest lies in the field of composite structures and finite element analysis.

In International Journals:

1. S. S. Sahoo, S. K. Panda and V. K. Singh "Experimental and numerical investigation of static and free vibration responses of woven Glass/Epoxy laminated composite plate," Journal of Materials: Design and Applications DOI: 10.1177/1464420715600191 2015.
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3. S. S. Sahoo and S. K. Panda "Static, Free Vibration and Transient Response of Laminated Composite Curved Panel-An Experimental Approach," European Journal of Mechanics/A (In press).
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7. V. K. Singh, S. S. Sahoo and S. K. Panda "Nonlinear vibration analysis of laminated carbon/epoxy curved shell panels using different shear deformation theory: Experimental validation," Applied Mathematics and Computation (Communicated).
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1. S. S. Sahoo, S. K. Panda and V. K. Singh, "Experimental and simulation study of flexural behaviour of woven Glass/Epoxy laminated composite plate." IOP Conf. Series: Materials Science and Engineering 75 (2015) doi:10.1088/1757-899X/75/1/012017 (4^{th} National Conference on Processing and Characterization of Materials (NCPCM 2014), December 5-6, 2014, NIT Rourkela).

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3. S. S. Sahoo, C. K. Hirwani and S. K. Panda, "Flexural and free vibration response of carbon/epoxy laminated plate using higher order theory- experimental analysis," International Conference on Computer Aided Engineering, December 10-12, 2015, GITAM, Hyderabad, India.

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